Matrices

$$\boxed{\cancel{\log(3+4i) = \log 5 + i\left(2n\pi + \tan^{-1}\left(\frac{4}{3}\right)\right)}}$$

Matrices

$$A = \begin{bmatrix} 2+3i & 1 & 1+3i \\ 4-i & 2i & 4+5i \\ 1+i & 3+4i & 5i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2-3i & 1 & 1-3i \\ 4+i & -2i & 4-5i \\ 1-i & 3-4i & -5i \end{bmatrix}$$

$$(\bar{A})^T = A^\theta = \begin{bmatrix} 2-3i & 4+i & 1-i \\ 1 & -2i & 3-4i \\ 1-3i & 4-5i & -5i \end{bmatrix}$$

Symmetric matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$A^T = A$

$\therefore A^T + A =$ symmetric

skew symmetric

$A^T = -A$

$A^T - A =$ skew sym

$$\boxed{A = \frac{1}{2}\left(A + \bar{A}^T\right) + \frac{1}{2}\left(A - \bar{A}^T\right)}$$

Hermition matrix

$$a_{ij} = \overline{a_{ji}}$$

$$A + A^{\theta} = \text{Hermition matrix}$$

skew Hermition matrix

$$a_{ij} = -\overline{a_{ji}}$$

$$A - A^{\theta} = \text{skew Hermitian matrix}$$

$$\boxed{A = \frac{1}{2}\left(A + A^{\theta}\right) + \frac{1}{2}\left(A - A^{\theta}\right)}$$

Qu Express A ~~as~~ matrix A as sum of Hermition and skew Hermition matrix

$$A = \begin{bmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix}$$

$$A = \frac{1}{2}\left(A + A^{\theta}\right) + \frac{1}{2}\left(A - A^{\theta}\right)$$

$$\bar{A} = \begin{bmatrix} 2-3i & 2 & -3i \\ 2i & 0 & 1-2i \\ 4 & 2-5i & i \end{bmatrix}$$

$$A^{\theta} = \begin{bmatrix} 2-3i & 2i & 4 \\ 2 & 0 & 2-5i \\ -3i & 1-2i & i \end{bmatrix}$$

$$A + A^{\theta} = \begin{bmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix} + \begin{bmatrix} 2-3i & 2i & 4 \\ 2 & 0 & 2-5i \\ -3i & 1-2i & i \end{bmatrix}$$

$$A + A^{\theta} = \begin{bmatrix} 4 & 2+2i & 4+3i \\ 2-2i & 0 & 3-3i \\ 4-3i & 3+3i & 0 \end{bmatrix} = \cancel{\text{symmetric}}\ \text{Harmition}$$

$$A - A^{\theta} = \begin{bmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix} - \begin{bmatrix} 2-3i & 2i & 4 \\ 2 & 0 & 2-5i \\ -3i & 1-2i & i \end{bmatrix}$$

$$= \begin{bmatrix} 6i & 2-2i & -4+3i \\ -2-2i & 0 & -1+7i \\ 4+3i & 1+7i & -2i \end{bmatrix} = \text{Skew}\ \cancel{\text{symmetric}}\ \text{Harmition}$$

$$A = \frac{1}{2}(A + A^{\theta}) + \frac{1}{2}(A + A^{\theta})$$

$$A = \frac{1}{2}\begin{bmatrix} 4 & 2+2i & 4+3i \\ 2-2i & 0 & 3-3i \\ 4-3i & 3+3i & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 6i & 2-2i & -4+3i \\ -2-2i & 0 & -1+7i \\ 4+3i & 1+7i & -2i \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 4+6i & 4 & 6i \\ -4i & 0 & 2+4i \\ 8 & 4+10i & -2i \end{bmatrix}$$

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(1) Orthogonal matrix

A real square matrix is said to be orthogonal if $AA^T = A^TA = I$

If A is orthogonal

$$A^{-1} = A^T$$

Unitary matrix

A square matrix A is said to be unitary matrix If $AA^{\theta} = A^{\theta}A = I$

If A is untary $A^{-1} = A^{\theta}$

Normal Form

every matrix can be reduced to $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ Form by elementary transformation

r is rank of A

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

Que P.T. $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is orthogonal & find A^{-1}

$$A^T = \frac{1}{3}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$AA^T = \frac{1}{3}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & -2-2+4 & -2+4-2 \\ -2-2+4 & +4+1+4 & +4-2-2 \\ -2+4-2 & 4-2-2 & 4+4+1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$AA^T = I$

$\therefore$ A is orthogonal matrix

$A^T = A^{-1}$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$

PT $A = \frac{1}{3}\begin{bmatrix} 2+i & 2i \\ 2i & 2-i \end{bmatrix}$ is unitary

$$AA^{\theta} = I$$

$$\bar{A} = \frac{1}{3}\begin{bmatrix} 2-i & -2i \\ -2i & 2+i \end{bmatrix}$$

$$A^{\theta} = (\bar{A})^{T} = \frac{1}{3}\begin{bmatrix} 2-i & -2i \\ -2i & 2+i \end{bmatrix}$$

$$AA^{\theta} = \frac{1}{3}\begin{bmatrix} 2+i & 2i \\ 2i & 2-i \end{bmatrix} \frac{1}{3}\begin{bmatrix} 2-i & -2i \\ -2i & 2+i \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 4+1-4i^2 & \cancel{-4i-2i^2+4i+2i^2} \\ \cancel{4i-2i^2-4i+2i^2} & -4i^2+4+1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = A^{\theta}$$

$$\bar{A}^{+} = \frac{1}{3}\begin{bmatrix} 2-i & -2i \\ -2i & 2+i \end{bmatrix}$$

Que Reduce the matrix A transform to norma Find its rank

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

A : $R_2 \to R_2 - R_1$ $R_3 \to R_3 - 3R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix}$$

$C_2 \to C_2 - C_1$ $C_3 \to C_3 - C_1$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$ $R_2 \to -\frac{1}{2} R_2$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - C_2$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A \sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

is normal form

rank = 2

1) Prove that following matrix is orthogonal and Hence find A^{-1}

$$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

$$A^T = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$AA^T = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} +4+1+4 & -4+2+2 & -2-2+4 \\ -4+2+2 & 4+4+1 & 2-4+2 \\ -2-2+4 & 2-4+2 & 1+4+4 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$AA^T = \frac{9}{9}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{AA^T = I}$$

$\therefore$ A is orthogonal matrix.

$\therefore A^{-1} = A^T$

$$\boxed{A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}}$$

(2) If $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal find a, b, c.

given matrix is orthogonal

$\therefore AA^T = I$

$$A^T = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ a & b & c \end{bmatrix}$$

$$I = AA^T = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ a & b & c \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

by equality.

$$\frac{1}{9} + \frac{4}{9} + a^2 = 1$$

$$\frac{5}{9} + a^2 = 1$$

$$a^2 = 1 - \frac{5}{9}$$

$$a^2 = \frac{4}{9}$$

$$a = \sqrt{\frac{4}{9}}$$

$$\boxed{a = \pm \frac{2}{3}}$$

$$\frac{4}{9} + \frac{1}{9} + b^2 = 1$$

$$b^2 = 1 - \frac{5}{9}$$

$$b^2 = \frac{4}{9}$$

$$\boxed{b = \pm \frac{2}{3}}$$

$$\frac{4}{9} + \frac{4}{9} + c^2 = 1$$

$$\frac{8}{9} + c^2 = 1$$

$$c^2 = 1 - \frac{8}{9}$$

$$c^2 = \frac{1}{9}$$

$$\boxed{c = \pm \frac{1}{3}}$$

Que. symm show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew symmetric matrix

let A be any square matrix

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

assume $A = P + Q$

$$P = \frac{1}{2}(A + A^T)$$

$$P^T = \frac{1}{2}(A + A^T)^T$$

$$P^T = \frac{1}{2}\left[(A^T) + (A^T)^T\right]$$

$$P^T = \frac{1}{2}\left[A^T + A\right]$$

$$P^T = \frac{1}{2}(A + A^T)$$

$$P^T = P$$

$\therefore$ P is a symmetric matrix

$$Q = \frac{1}{2}(A - A^T)$$

$$Q^T = \frac{1}{2}(A - A^T)^T$$

$$Q^T = \frac{1}{2}\left[(A^T) - (A^T)^T\right]$$

$$Q^T = \frac{1}{2}(A^T - A)$$

$$Q^T = -\frac{1}{2}(-A^T + A)$$

$$Q^T = -\frac{1}{2}(A - A^T)$$

$$Q^T = -Q$$

$\therefore$ Q is a skew symmetric

Thus we have expressed A as the sum of a symmetric and skew-symmetric matrix

To prove uniqueness ~~let~~ R

let $A = R + S$

R is a symmetric matrix

S is a skew symmetric matrix

$$R^T = R$$

$$S^T = -S$$

$$A^T = R^T + S^T$$

$$\boxed{A^T = R - S}$$

$$P = \frac{1}{2}(A + A^T)$$

$$P = \frac{1}{2}(R + \cancel{S} + R - \cancel{S})$$

$$P = \frac{1}{\cancel{2}}(\cancel{2}R)$$

$$\boxed{P = R}$$

$$Q = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}(\cancel{R} + S - \cancel{R} + S)$$

$$= \frac{1}{\cancel{2}}(\cancel{2}S)$$

$\boxed{Q = S}$ Hence representation $A = P + Q$ is unique

Que $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$ is orthogonal Find A^{-1}

$$A^T = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

For orthogonal $AA^T = I$

$$AA^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} 64+16+1 & -8+16-8 & -32+28+4 \\ -8+16-8 & 1+16+64 & 4+28-32 \\ -32+28+4 & 4+28-32 & 16+49+16 \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix}$$

$$AA^T = \frac{\cancel{81}}{\cancel{81}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{AA^T = I}$$

hence A is the orthogonal matrix

$$\therefore \quad A^{-1} = A^{T}$$

$$\boxed{A^{-1} = \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}}$$

Que Prove that matrix $A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is unitary and A^{-1}.

$$\bar{A} = \frac{1}{2}\begin{bmatrix} \sqrt{2} & i\sqrt{2} & 0 \\ -i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^{\theta} = (\bar{A})^{T} = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$AA^{\theta} = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix} \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2+2+0 & -2i+2i+0 & 0+0+0 \\ 2i-2i+0 & 2+2+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+4 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \frac{4}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AA^{\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AA^{\theta} = I$$

$\therefore$ A is unitary matrix

$$A^{-1} = A^{\theta}$$

$$\boxed{A^{-1} = \begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}}$$

Que Find Rank of ~~pa~~ matrix

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \qquad R_1 \to R_1 - R_2$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \qquad R_2 \to R_2 - 2R_1$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \qquad C_2 \to C_2 + C_3$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \qquad C_3 \to C_3 - 4C_2$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = I_3$$

$$\boxed{\therefore\ r(A) = 3}$$

Q) Find the Rank of matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 \to R_2 - 2R_1$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C_2 \to C_2 - C_1$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C_3 \leftrightarrow C_2$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad R_3 \leftrightarrow R_2$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\boxed{\therefore \ r(A) = 2}$$

$$\log(\ldots) = 2n\pi i + i\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)$$

Qu. Express matrix A as sum of symmetric and skew symmetric matrix

$$A = \begin{bmatrix} 6 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 4 \\ 5 & 1 & 5 \\ -3 & 1 & 9 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 4 \\ 5 & 1 & 5 \\ -3 & 1 & 9 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 0 & 6 & 1 \\ 6 & 2 & 6 \\ 1 & 6 & 18 \end{bmatrix}$$ = symmetric matrix

$$A - A^T = \begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 4 \\ 5 & 1 & 5 \\ -3 & 1 & 9 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 0 & 4 & -7 \\ -4 & 0 & -4 \\ -7 & 4 & 0 \end{bmatrix}$$ = skew symmetric matrix

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$$A = \frac{1}{2}\begin{bmatrix} 0 & 6 & 1 \\ 6 & 2 & 6 \\ 1 & 6 & 18 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 4 & -7 \\ -4 & 0 & -4 \\ 7 & 4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 3 & \frac{1}{2} \\ 3 & 1 & 3 \\ \frac{1}{2} & 3 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -\frac{7}{2} \\ -2 & 0 & -2 \\ \frac{7}{2} & 2 & 0 \end{bmatrix}$$

symmetric matrix   skew symmetric matrix

Ques Express matrix A as sum of Hermitian and skew symmetric matrix

$$A = \begin{bmatrix} 2+3i & 2 & 3i \\ 2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix}$$

$$A = \frac{1}{2}(A + A^{\theta}) + \frac{1}{2}(A + A^{\theta})$$

$$\bar{A} = \begin{bmatrix} 2-3i & 2 & -3i \\ -2i & 0 & 1-2i \\ 4 & 2-5i & i \end{bmatrix}$$

$$(\bar{A})^T = A^{\theta} = \begin{bmatrix} 2-3i & -2i & 4 \\ 2 & 0 & 2-5i \\ -3i & 1-2i & i \end{bmatrix}$$

$$A + A^{\theta} = \begin{bmatrix} 2+3i & 2 & 3i \\ 2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix} + \begin{bmatrix} 2-3i & -2i & 4 \\ 2 & 0 & 2-5i \\ -3i & 1-2i & i \end{bmatrix}$$

$$A + A^{\theta} = \begin{bmatrix} 4 & 2\mp 2i & 4+3i \\ 2+2i & 0 & 3-3i \\ 4-3i & 3+3i & 0 \end{bmatrix} = \text{Hermitian matrix}$$

$$A \mp A^{\theta} = \begin{bmatrix} 2+3i & 2 & 3i \\ 2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix} - \begin{bmatrix} 2-3i & -2i & 4 \\ 2 & 0 & 2-5i \\ -3i & 1-2i & i \end{bmatrix}$$

$$A - A^{\theta} = \begin{bmatrix} 6i & 2+2i & -4+3i \\ -2+2i & 0 & -1+7i \\ 4+3i & 1+7i & -2i \end{bmatrix} = \text{skew Hermitian matrix}$$

$$A = \frac{1}{2}(A + A^{\theta}) + \frac{1}{2}(A - A^{\theta})$$

$$A = \frac{1}{2}\begin{bmatrix} 4 & 2-2i & 4+3i \\ 2+2i & 0 & 3-3i \\ 4-3i & 3+3i & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 6i & 2+2i & -4+3i \\ -2+2i & 0 & -1+7i \\ 4+3i & 1+7i & -2i \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1-i & 2+\frac{3}{2}i \\ 1+i & 0 & \frac{3}{2}-\frac{3}{2}i \\ 2-\frac{3}{2}i & \frac{3}{2}+\frac{3i}{2} & 0 \end{bmatrix} + \begin{bmatrix} 3i & 1+i & -2+\frac{3}{2}i \\ -1+i & 0 & \frac{-1+7i}{2} \\ \frac{4+3i}{2} & \frac{1+7i}{2} & -i \end{bmatrix}$$

= Hermitian matrix + skew Hermitian matrix

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Que) Prove that every skew Hermitian matrix A can be written as B + iC where B is a real skew symmetric matrix and C is real symmetric matrix

$$A = B + iC$$

since A is skew Hermitian

$$A^{\theta} = -A$$

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$z + \bar{z} = 2x$$

$$z - \bar{z} = 2iy$$

$$B = x = \frac{1}{2}(z + \bar{z}) \qquad C = y = \frac{1}{2i}(z - \bar{z})$$

$$A = \frac{1}{2}(A + \bar{A}) + i \cdot \frac{1}{2i}(A - \bar{A}) = B + iC$$

$$B = \frac{1}{2}(A + \bar{A}) \quad \text{real}$$

$$C = \frac{1}{2}(A - \bar{A}) \quad \text{real}$$

$$\cancel{\bar{A} = A^T} \qquad A^T = -A$$

$$B = \frac{1}{2}(A + \bar{A})$$

$$B^T = \frac{1}{2}(A^T + \bar{A}^T)$$

$$B^T = \frac{1}{2}(A^T + A^{\theta}) \qquad A^{\theta} = -A$$

$$B = \frac{1}{2}\left((-A^{\theta})^{T} + (-A)\right)$$

$$B^{T} = \frac{1}{2}\left((-(\bar{A})^{T})^{T} - A\right)$$

$$= \frac{1}{2}\left(-\bar{A} - A\right)$$

$$\boxed{B^{T} = -\frac{1}{2}\left(A + \bar{A}\right)}$$

Hence $B^{T} = -B$

Hence B is skew symmetric matrix

$$C^{T} = \frac{1}{2}\left(A - \bar{A}\right)$$

$$C^{T} = \frac{1}{2}\left(A^{T} - (\bar{A})^{T}\right)$$

$$C^{T} = \frac{1}{2}\left(A^{T} - A^{\theta}\right)$$

$$= \frac{1}{2}\left[(-A^{\theta})^{T} - (-A)\right]$$

$$= \frac{1}{2}\left[(-(\bar{A}^{T})^{T}) + A\right]$$

$$= \frac{1}{2}\left[-\bar{A} + A\right]$$

$$\boxed{C^{T} = \frac{1}{2}\left[A - \bar{A}\right]}$$

$$\boxed{C^{T} = C}$$

Hence C is real symmetric matrix.

① Express skew Hermitian matrix A as $P + iQ$ where P is real skew symmetric and Q is real symmetric matrix

$$A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & -3i \\ -1-i & 3i & 0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} -2i & 2-i & 1+i \\ -2-i & i & -3i \\ -1+i & -3i & 0 \end{bmatrix}$$

$$A + \bar{A} = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix} + \begin{bmatrix} -2i & 2+i & 1+i \\ -2-i & i & -3i \\ -1+i & -3i & 0 \end{bmatrix}$$

$$A + \bar{A} = \begin{bmatrix} 0 & 2 & 2 \\ -4 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

$$A - \bar{A} = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix} - \begin{bmatrix} -2i & 2-i & 1+i \\ -2+i & i & -3i \\ -1+i & -3i & 0 \end{bmatrix}$$

$$A - \bar{A} = \begin{bmatrix} 4i & 2i & -2i \\ 2i & -2i & 6i \\ -2i & 6i & 0 \end{bmatrix}$$

$$A = \frac{1}{2}(A + \bar{A}) + \frac{1}{2}(A - \bar{A})$$

$$A = \frac{1}{2}\begin{bmatrix} 0 & 4 & 2 \\ -4 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 4i & 2i & -2i \\ 2i & -2i & 6i \\ -2i & 6i & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2i & i & -i \\ i & -i & 3i \\ -i & 3i & 0 \end{bmatrix}$$

P = skew symmetric matrix Q = ~~ske~~ symmetric matrix

Que Find a, b, c if A is orthogonal when A

$$A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$$

Since A is orthogonal

$\therefore AA^T = I$

$$A^T = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ a & b & c \end{bmatrix}$$

$$AA^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix} \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ a & b & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{81}\begin{bmatrix} 64+16+a^2 & -8+16+ab & -32+28+ac \\ -8+16+ab & 1+16+b^2 & 4+28+bc \\ -32+28+ac & 4+28+bc & 16+49+c^2 \end{bmatrix}$$

$$\frac{1}{81}\begin{bmatrix} 80+a^2 & 8+ab & -4+ac \\ 8+ab & 17+b^2 & 32+bc \\ -4+ac & 32+bc & 65+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 80+a^2 & 8+ab & -4+ac \\ 8+ab & 17+b^2 & 32+bc \\ -4+ac & 32+bc & 65+c^2 \end{bmatrix} = \begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix}$$

$80+a^2 = 81$ $\qquad$ $8+ab = 0$ $\qquad$ $-4+ac = 0$

$17+b^2 = 81$ $\qquad$ $32+bc = 0$ $\qquad$ $65+c^2 = 81$

$a^2 = 1$ $\qquad$ $b^2 = 81-17$ $\qquad$ $c^2 = 81-65$

$\boxed{a = \pm 1}$ $\qquad$ $b^2 = 64$ $\qquad$ $c^2 = 16$

$\boxed{b = \pm 8}$ $\qquad$ $\boxed{c = \pm 4}$

Que Show that the matrix $A = \begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is unitry matrix if $\alpha^2+\beta^2+\gamma^2+\delta^2 = 1$

$$A = \begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} \alpha-i\gamma & -\beta-i\delta \\ \beta-i\delta & \alpha+i\gamma \end{bmatrix}$$

$$(\bar{A})^T = A^{\theta} = \begin{bmatrix} \alpha-i\gamma & \beta-i\delta \\ -\beta-i\delta & \alpha+i\gamma \end{bmatrix}$$

$$A A^{\theta} = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix} \begin{bmatrix} \alpha - i\gamma & \beta - i\delta \\ -\beta - i\delta & \alpha + i\gamma \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \gamma^2 + \beta^2 + \delta^2 & (\alpha + i\gamma)(\beta - i\delta) + (-\beta + i\delta)(\alpha + i\gamma) \\ (\beta + i\delta)(\alpha - i\gamma) + (\alpha - i\gamma)(-\beta - i\delta) & \beta^2 + \delta^2 + \alpha^2 + \gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \gamma^2 + \beta^2 + \delta^2 & (\alpha + i\gamma)(\beta - i\delta - \beta + i\delta) \\ (\alpha - i\gamma)(\beta + i\delta - \beta - i\delta) & \beta^2 + \delta^2 + \alpha^2 + \gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \gamma^2 + \beta^2 + \delta^2 & 0 \\ 0 & \beta^2 + \delta^2 + \alpha^2 + \gamma^2 \end{bmatrix}$$

if $\alpha^2 + \gamma^2 + \beta^2 + \delta^2 = 1$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

$\therefore$ A is a unitary matrix.

Reduction of matrix A to normal Form PAQ

If A is matrix $m \times n$ of rank r then there exists ~~tot~~ non singular matrices P and Q such that PAQ is in normal For $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = PAQ$

① If A is $m \times n$ matrix

$A_{m \times n} = I_m A I_n$

$A_{3 \times 4} = I_3 A I_4$

$a_{11} = 1$

any row operation are performed on A same operation same on prefactor

$R_2 \to R_2 - R_1$

any column operation are performed on A same on prefactor I_n

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = PAQ$$

$$A^{-1} = QP$$

Que. Find non singular matrices P and Q such that PAQ is in normal form and hence find its rank.

$$A = \begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$$

$$A_{3\times4} = I_3 \, A \, I_4$$

$$\begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_1 \leftrightarrow C_3$

$$\begin{bmatrix} 1 & 3 & 4 & 6 \\ 2 & 4 & 2 & 2 \\ 5 & 14 & 12 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1 \qquad R_3 \to R_3 - 5R_1$

$$\begin{bmatrix} 1 & 3 & 4 & 6 \\ 0 & -2 & -6 & -10 \\ 0 & -1 & -8 & -14 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 6 \\ 0 & -2 & -6 & -10 \\ 0 & -1 & -8 & -14 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - 3C_1, \quad C_3 \to C_3 - 4C_1, \quad C_4 \to C_4 - 6C_1$

$$\begin{bmatrix}1&0&0&0\\0&-2&-6&-10\\0&-1&-8&-14\end{bmatrix}=\begin{bmatrix}1&0&0\\-2&1&0\\-5&0&1\end{bmatrix}A\begin{bmatrix}0&0&1&0\\0&1&\cancel{-9}&0\\1&-3&-4&-6\\0&0&0&1\end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix}1&0&0&0\\0&-1&-8&-14\\0&-2&-6&-10\end{bmatrix}=\begin{bmatrix}1&0&0\\-5&0&1\\-2&1&0\end{bmatrix}A\begin{bmatrix}0&0&1&0\\0&1&0&0\\1&-3&-4&-6\\0&0&0&1\end{bmatrix}$$

$R_2 \to R_2 \times -1$

$$\begin{bmatrix}1&0&0&0\\0&1&8&14\\0&-2&-6&-10\end{bmatrix}=\begin{bmatrix}1&0&0\\5&0&-1\\-2&1&0\end{bmatrix}A\begin{bmatrix}0&0&1&0\\0&1&0&0\\1&-3&-4&-6\\0&0&0&1\end{bmatrix}$$

$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix}1&0&0&0\\0&1&8&14\\0&0&10&18\end{bmatrix}=\begin{bmatrix}1&0&0\\5&0&-1\\8&1&-2\end{bmatrix}A\begin{bmatrix}0&0&1&0\\0&1&0&0\\1&-3&-4&-6\\0&0&0&1\end{bmatrix}$$

$-6+48$

$C_3 \to C_3 - 8C_2$ $\quad$ $C_4 \to C_4 - 14C_2$

$-4+24$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&10&18\end{bmatrix}=\begin{bmatrix}1&0&0\\5&0&-1\\8&1&-2\end{bmatrix}A\begin{bmatrix}0&0&1&0\\0&1&-8&-14\\1&-3&20&36\\0&0&0&1\end{bmatrix}$$

$R_3 \to \frac{R_3}{10}$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&18\end{bmatrix}=\begin{bmatrix}1&0&0\\5&0&-1\\8&1&-2\end{bmatrix}A\begin{bmatrix}0&0&1/10&0\\0&1&-8/10&-14\\1&-3&2&36\\0&0&0&1\end{bmatrix}$$

$C_4 \to C_4 - 18C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 0 & -1 \\ 8 & 1 & -2 \end{bmatrix} A \begin{bmatrix} 0 & 0 & \frac{1}{10} & -\frac{9}{5} \\ 0 & 1 & -\frac{4}{5} & \frac{2}{5} \\ 1 & -3 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix} = PAQ \text{ is normal form}$$

$$r(A) = 3$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 0 & -1 \\ 8 & 1 & -2 \end{bmatrix} \qquad Q = \begin{bmatrix} 0 & 0 & \frac{1}{10} & -\frac{9}{5} \\ 0 & 1 & -\frac{4}{5} & \frac{2}{5} \\ 1 & -3 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ques Find non singular matrices P and Q such that PAQ is in normal form and hence find its rank and A^{-1}

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$A_{3\times3} = I_3 \, A \, I_3$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -6 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to R_2 - 2C_1$, $C_3 \to C_3 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -6 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$ $R_2 \to -R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 - 6C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to \frac{R_3}{-4}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix} A \begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$I_3 = PAQ$

$r(A) = 3$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = QP$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \frac{1}{4} \begin{bmatrix} 4 & 0 & 0 \\ 8 & -4 & 0 \\ 2 & -1 & -1 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4-16+18 & 0+8-9 & 0+0-9 \\ 0+8-12 & 0-4+6 & 0+0+6 \\ 0+0+2 & 0+0-1 & 0+0-1 \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{1}{4} \begin{bmatrix} 6 & -1 & -9 \\ -4 & 2 & 6 \\ 2 & -1 & -1 \end{bmatrix}}$$

Linear Equation

$ax + by + cz = d$

Non homogeneous linear equation
consider the equations

$a_{11}x_1 + a_{12}x_2 \cdots + a_{1n}x_n = d_1$

$a_{21}x_1 + a_{22}x_2 \cdots + a_{2n}x_n = d_2$

$a_{m1}x_1 + a_{m2}x_2 \cdots + a_{mn}x_n = d_m$

$$A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$$ - Coeffient matrix unkown variables

$$B = \begin{bmatrix} d_1 \\ d_2 \\ d_m \end{bmatrix}$$ - ~~Coefficient~~ constant matrix

$[A, B]$ is formed by coefficient matrix and ~~coefficient~~ constant matrix is called Augmented matrix

① we write $AX = B$

② we apply row operations on A and B to reduce in echelon form

rank of A is -

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 1 \end{bmatrix} \quad r(A) = 3$$

$$(A, B) = \left[\begin{array}{ccc:c} 1 & 2 & 3 & 2 \\ 0 & 2 & 5 & 5 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

$r(A) = 3$, $r(A, B) = 3$

① if $r(A) < r(A, B)$
system is in consistent it has no solution

② If ~~rf~~ $r(A) = r(A, B)$
system is consistent it has solution

If $r = n$ i.e rank of A is equal to no of unkown varible it has unique solution
$x = 1$, $y = 2$, $z = 3$

if $r < n$ ie ~~rant~~ then it has infinite solution
$n = 3$, $r = 2$
$n - r = 3 - 2 = 1$ · - is called arbitory constant

Que Test the consistency solve them if consistent

$2x - 3y + 7z = 5$

$3x + y - 3z = 13$

$2x + 19y - 47z = 32$

Sol^n $Ax = B$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$R_1 \to -R_1 + R_2$ $\quad R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 4 & -10 \\ 3 & 1 & -3 \\ 0 & 22 & -54 \end{bmatrix} x = \begin{bmatrix} 8 \\ 13 \\ 27 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 22 & -54 \end{bmatrix} x \begin{bmatrix} 8 \\ -11 \\ 27 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 8 \\ -11 \\ 5 \end{bmatrix}$$

$r(A) = 2$

$$(A, B) = \begin{bmatrix} 1 & 4 & -10 & : & 8 \\ 0 & -11 & 27 & : & -4 \\ 0 & 0 & 0 & : & 5 \end{bmatrix}$$

$r(A,B) = 3$

$r(A) < r(A,B) = 3$

system is inconsistent it has no solution.

Que $x - 2y + 3t = 2$

$2x + y + z + t = 4$

$4x - 3y + z + 7t = 8$

$AX = B$

$$\begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & 1 & 1 & 1 \\ 4 & -3 & 1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$$

$$(A,B) = \begin{bmatrix} 1 & -2 & 0 & 3 & : & 2 \\ 2 & 1 & 1 & 1 & : & 4 \\ 4 & -3 & 1 & 7 & : & 8 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & -2 & 0 & 3 & : & 2 \\ 0 & 5 & 1 & -5 & : & 0 \\ 0 & 5 & 1 & -5 & : & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & -2 & 0 & 3 & : & 2 \\ 0 & 5 & 1 & -5 & : & 0 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$r(A) = 2$, $r(A,B) = 2$

$$r(A) = r(A, B) = 2$$

system is consistent it has solution

$n = 4$, $r = 2$

$n - r = 4 - 2 = 2$ Paramaters

$$\begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 5 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$x - 2y + 3t = 0$

$5y + z - 5t = 0$

$y = t_2$

$r(A) < r(A, B)$

system is inconsistent it has no sol^n

$r(A) = r(A, B)$

consistent it has sol^n

Matrices

Reduction of a matrix A to Normal Form PAQ

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$$A = I_3 A I_3$$

Que If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ Find two matrices P and Q such that PAQ is in normal form also find Rank of A.

$$A = I_m A I_n$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1 \quad , \quad C_3 \to C_3 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{R_2}{-2}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to 2R_3 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank of matrix A = 2

$\sigma(A) = 2$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 2 & -1 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

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Normal Form PAQ

Find non singular matrices P and Q such that PAQ is normal Form Also Find Rank of A and A^{-1}

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$A = I_3 A I_3$

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$ $\quad$ $C_3 \to C_3 + 2C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{R_2}{5}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1. \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\cancel{R}$ $C_3 \to C_3 + \frac{2}{5} C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 6/5 \\ 0 & 1 & 2/5 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \cancel{\to} 5 \times R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 2 & 5 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 6/5 \\ 0 & 1 & 2/5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 2 & 5 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 6/5 \\ 0 & 1 & 2/5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 2 & 5 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & -2 & 6/5 \\ 0 & 1 & 2/5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{r(A) = 3}$$

$I = PAQ$

$A^{-1} = \cancel{PQ}$

$P^{-1} = AQ$

$A^{-1}P^{-1} = Q$

$$\boxed{A^{-1} = QP}$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 6/5 \\ 0 & 1 & 2/5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{2}{5}+\frac{12}{5} & 0-\frac{2}{5}+\frac{12}{5} & 0+0+6 \\ 0+\frac{1}{5}+\frac{4}{5} & 0+\frac{1}{5}+\frac{4}{5} & 0+0+2 \\ 0+0+2 & 0+0+2 & 0+0+5 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}}$$

* Linear Dependence and Linear independence

$k_1x_1 + k_2x_2 + k_3x_3 + \cdots + k_nx_n = 0$

if $k_1, k_2, k_3, k_4 \ldots k_n$ are not equal to zero Then it is called Linear Dependence

if $k_1 = k_2 = k_3 = k_4 + \cdots + k_n = 0$ then it is called Linear independence

Que Determine the linear dependence or independence of vector $(2,-1,3,2)$, $(1,3,4,2)$, $(3,-5,2,2)$. Find the relation betn them if dependent

$$k_1 x_1 + k_2 x_2 + k_3 x_3 = 0$$

$$k_1 [2,-1,3,2] + k_2 [1,3,4,2] + k_3 [3,-5,2,2] = 0$$

$$2k_1 + k_2 + 3k_3 = 0$$
$$-k_1 + 3k_2 - 5k_3 = 0$$
$$3k_1 + 4k_2 + 2k_3 = 0$$
$$2k_1 + 2k_2 + 2k_3 = 0$$

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 3 & -5 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \cancel{k_4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} -1 & 3 & -5 \\ 2 & 1 & 3 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to -R_1$

$$\begin{bmatrix} 1 & -3 & 5 \\ 2 & 1 & 3 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0$$

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$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$ $R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 5 \\ 0 & 7 & -7 \\ 0 & 13 & -13 \\ 0 & 8 & -8 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0$$

$\frac{R_2}{7}$

$$\begin{bmatrix} 1 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 13 & -13 \\ 0 & 8 & -8 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0$$

$R_3 \to R_3 - 13R_2$, $R_4 \to R_4 - 8R_2$

$$\begin{bmatrix} 1 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} k_1 - 3k_2 + 5k_3 \\ k_2 - k_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$k_1 - 3k_2 + 5k_3 = 0$

$k_2 - k_3 = 0$

$k_2 = k_3$

Put $\boxed{k_3 = t}$

$\boxed{k_2 = t}$

$$k_1 - 3k_2 + 5k_3 = 0$$
$$k_1 - 3(t) + 5t = 0$$

$$k + 2t = 0$$

$$\boxed{k = -2t}$$

k_1, k_2, k_3 are not zero

$\therefore$ given vector are linear dependence

If vectors are dependent

$$k_1 x_1 + k_2 x_2 + k_3 x_3 = 0$$
$$-2t\, x_1 + t x_2 + t x_3 = 0$$

$$t(-2x_1 + x_2 + x_3) = 0$$

$$-2x_1 + x_2 + x_3 = 0$$

$$2x_1 - x_2 - x_3 = 0$$
$$2x_1 = x_2 + x_3$$

$$\boxed{x_1 = \frac{x_2 + x_3}{2}}$$

This is the relation betn the vectors.

Que Show that the following vectors are linearly independent

$x_1 = [1, 2, -1, 0]$, $x_2 = [1, 3, 1, 2]$, $x_3 = [4, 2, 1, 0]$

$x_4 = [6, 1, 0, 1]$

$$k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 = 0$$

$$k_1 [1, 2, -1, 0] + k_2 [1, 3, 1, 2] + k_3 [4, 2, 1, 0] + k_4 [6, 1, 0, 1] = 0$$

$$k_1 + k_2 + 4k_3 + 6k_4 = 0$$
$$2k_1 + 3k_2 + 2k_3 + k_4 = 0$$
$$-k_1 + k_2 + k_3 = 0$$
$$2k_2 + k_4 = 0$$

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 2 & 3 & 2 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $\quad R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 0 & 1 & -6 & -11 \\ 0 & 2 & 5 & 6 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$, $\quad R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 0 & 1 & -6 & -11 \\ 0 & 0 & 17 & 28 \\ 0 & 0 & 12 & 23 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$\frac{R_3}{17}$

$$\begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 1 & -6 & +11 \\ 0 & 0 & 1 & 28/17 \\ 0 & 0 & 12 & 23 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_4 \to R_4 - 12R_3$

$$\begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 1 & -6 & -11 \\ 0 & 0 & 1 & 28/17 \\ 0 & 0 & 0 & 55/17 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} k_1 + k_2 + 4k_3 + 5k_4 \\ + k_2 - 6k_3 - 11k_4 \\ + k_3 + \frac{28}{17}k_4 \\ \frac{55}{17}k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\frac{55}{17}k_4 = 0$

$\boxed{k_4 = 0}$

$k_3 + \frac{28}{17}k_4 = 0$

$k_3 + 0 = 0$

$\boxed{k_3 = 0}$

$k_2 - 6k_3 - 11k_4 = 0$

$k_2 - 0 - 0 = 0$

$\boxed{k_2 = 0}$

$k_1 + k_2 + 4k_3 + 5k_4 = 0$

$k_1 + 0 + 0 + 0 = 0$

$\boxed{k_1 = 0}$

$k_1 = k_2 = k_3 = k_4 = 0$

all value of k are zero

$\therefore$ all given vector are linearly independent.

* Condition of consistency.

~~Step~~ steps

I) $Ax = B$

II) Echelon Form

III) Rank $A <$ Rank $[A, B]$

Equations are inconsistent i.e. No solution

IV) Rank $A =$ Rank $[A, B]$

$\therefore$ Equation are consistent

$r = n$	$r < n$
unique soln	Infinite soln

Que $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$

$Ax = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \cancel{4}\ 1 \\ \cancel{6}\ 3 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \cancel{8}\ 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

$r(A) = 3$

$r(A, B) = 3$

$r(A) = r(A, B)$

$\therefore$ eq^n are consistent.

$r(A) = n$

$\therefore$ system has unique solution

$$\begin{bmatrix} x + y + z \\ y + 2z \\ 2z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$x + y + z = 3$

$y + 2z = 1$

$2z = 0$

$\boxed{z = 0}$, $y + 0 = 1$

$\boxed{y = 1}$

$x + 1 + 0 = 3$

$\boxed{x = 2}$