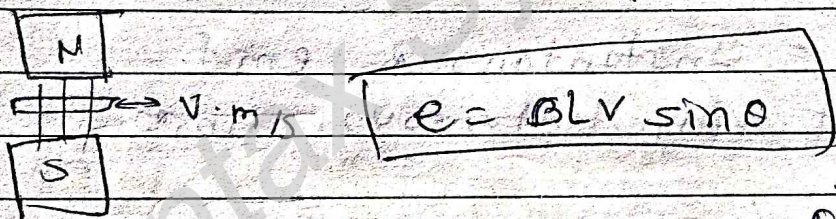
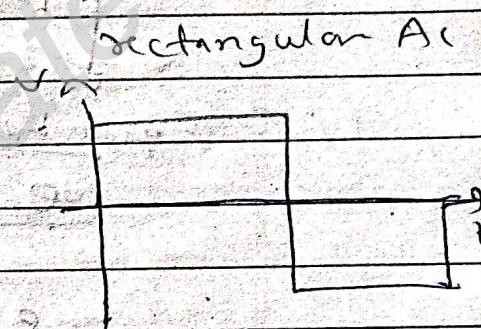
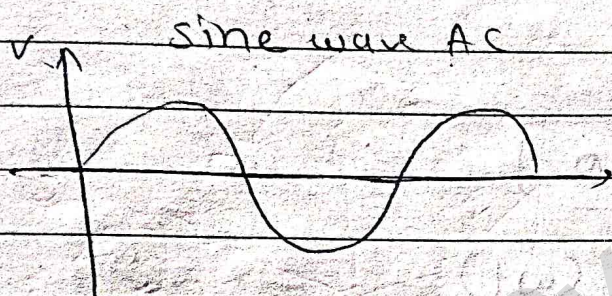
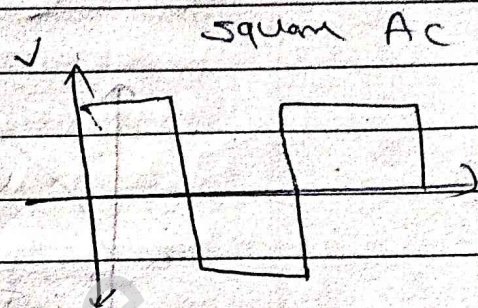
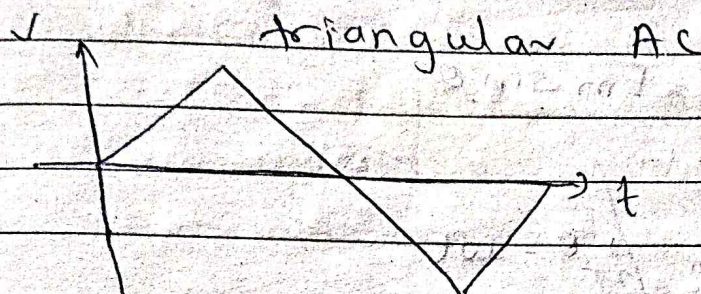


AC. circuits

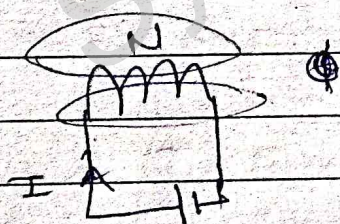


Generation :

I) A.C Fundamentals.



dynamically induce emf

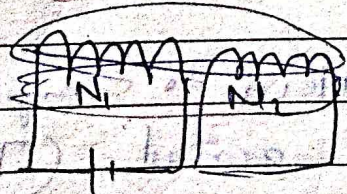


$$\psi = N \phi$$

$$e = \frac{d\psi}{dt}$$

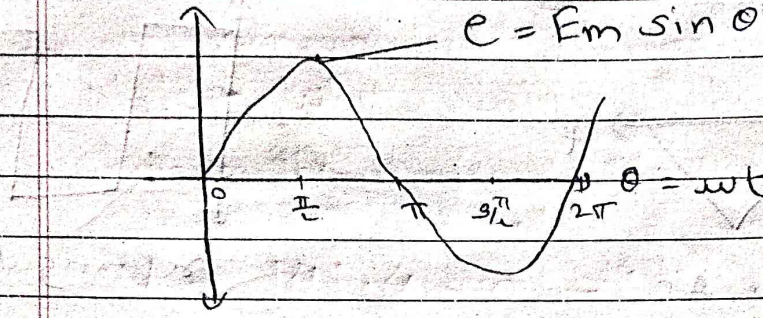
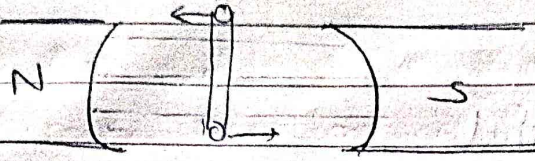
$$e = N \frac{d\phi}{dt}$$

it is statically induce emf



$$e_1 = N_1 \frac{d\phi}{dt}$$

$$e_2 = N_2 \frac{d\phi}{dt}$$



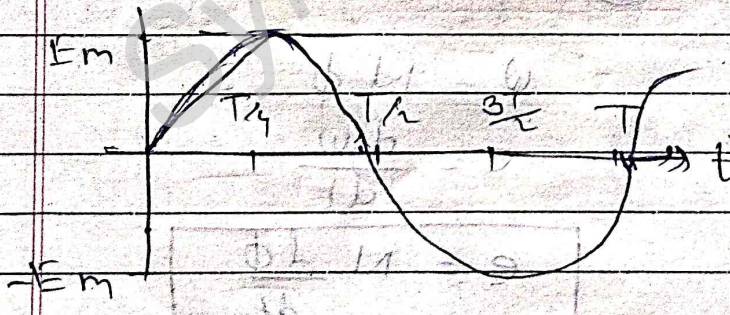
Equation

$$e = E_m \sin \theta$$

$$e = E_m \sin(\omega t)$$

$e =$ Instantaneous emf
 $t =$ Instantaneous time

Wave form



cycle - A complete set of positive and negative values of AC quantity is called as a cycle.

Time period - Time taken to complete one cycle is called time period. (T)

for $\forall x \in \mathbb{R} \quad \exists y \in \mathbb{R} \quad x + y = 0$ (2)

An AC voltage is given by $V = 141.4 \sin 314 t$
 determine ① time period, ② angular frequency
 ③ frequency ④ amplitude ⑤ value of
 instantaneous voltage at $t = 5 \text{ msec}$, $t = 15 \text{ msec}$
 $t = 7.5 \text{ msec}$

Note - while solving the problem on AC equation
 change the mode of the calculator to radian.

$$V = 141.4 \sin(314 t)$$

Compare with standard eqⁿ

$$V = V_m \sin \omega t$$

① $V_m = 141.4 \text{ V}$ ② $\omega = 314 \text{ rad/sec}$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{314}{2\pi}$$

③ $f = 49.97 \text{ Hz}$

$$T = \frac{1}{f}$$

$$T = \frac{1}{50} = 0.02 \text{ sec}$$

④ $T = 20 \text{ msec}$

⑤ $V = 141.4 \sin 314 \times t$

⑥ at $t = 5 \text{ msec} = 0.005$

$$V = 141.4 \sin(314 \times 0.005)$$

$$V = 141.4 \text{ V}$$

⑥ $t = 15 \text{ ms}$

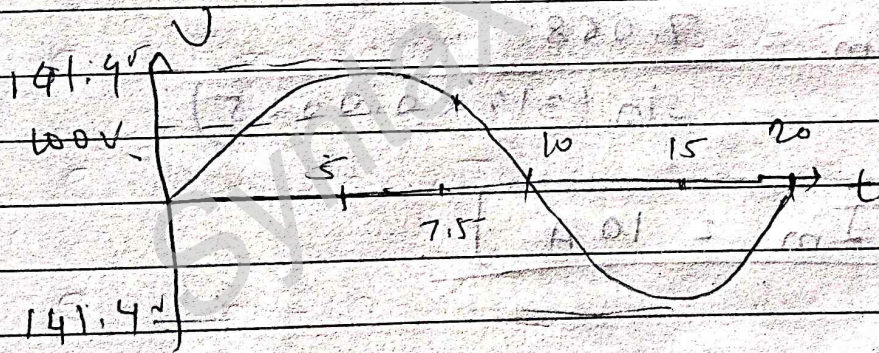
$$V = 141.4 \sin(314 \times 0.015)$$

$$V = -141.4 \text{ V}$$

⑦ $t = 7.5 \text{ msec}$

$$V = 141.4 \sin(314 \times 0.0075)$$

$$V = 100 \text{ V}$$



$$V = 100 \text{ V}$$

Therefore, the instantaneous value of voltage is 100 V.

Ques An AC current having 50 Hz frequency and 10 A amplitude takes 2.5 msec to reach maximum value 7.068 A write the eqn of the current in the standard form inserting all the values.

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 50 = 314.1 \text{ rad/sec}$$

$$i = I_m \sin(\omega t)$$

$$\text{when } t = 2.5 \times 10^{-3} \text{ s}, i = 7.068 \text{ A}$$

$$7.068 = I_m \sin(314 \times 2.5 \times 10^{-3})$$

$$I_m = 7.068$$

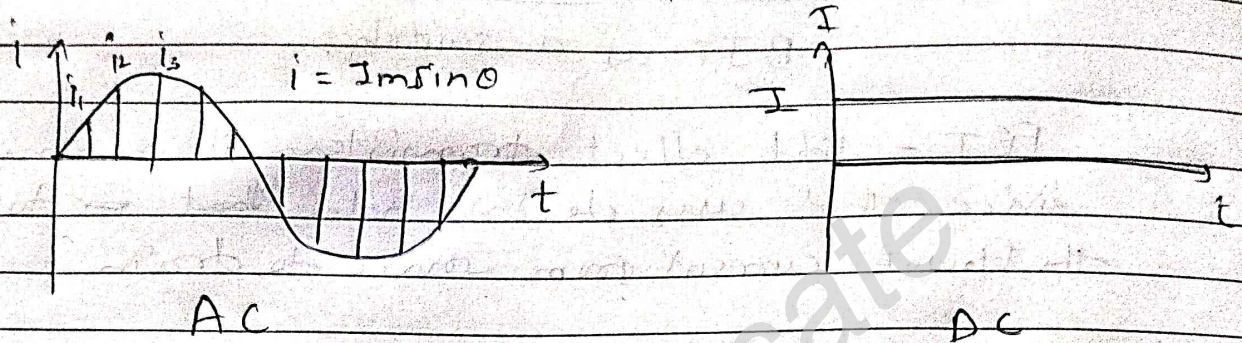
$$\sin(314 \times 0.0025)$$

$$I_m = 10 \text{ A}$$

$$i = 10 \sin(314 t)$$

This is the standard eqn of the current.

* Root mean square value (R_{ms}) - Effective value of AC current is a DC current which produces same heat in same resistance for same time



$$H_{ac} = \frac{i_1^2 R T/n}{J} + \frac{i_2^2 R T/n}{J} + \dots + \frac{i_n^2 R T/n}{J}$$

$$H_{dc} = \frac{I^2 R T}{J}$$

$$H_{dc} = H_{ac}$$

$$\frac{I^2 R T}{J} = \frac{i_1^2 R T/n}{J} + \frac{i_2^2 R T/n}{J} + \dots + \frac{i_n^2 R T/n}{J}$$

$$\frac{I^2 R T}{J} = \frac{R T}{J} \left[\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n} \right]$$

$$I^2 = \frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}$$

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}}$$

Called RMS value of current.

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + \dots + V_n^2}{n}}$$

$$i_{rms} = \sqrt{\frac{\int_0^{2\pi} i^2 d\theta}{2\pi}}$$

$$i = I_m \sin \theta$$

$$i = \sqrt{\frac{\int_0^{2\pi} I_m^2 \sin^2 \theta d\theta}{2\pi}}$$

$$i = \sqrt{\frac{I_m^2 \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta}{2\pi}}$$

$$i = \sqrt{\frac{I_m^2 \left[\int_0^{2\pi} 1 d\theta - \int_0^{2\pi} \cos 2\theta d\theta \right]}{4\pi}}$$

$$i = \sqrt{\frac{I_m^2 \left[2\pi - [\sin 2\theta]_0^{2\pi} \right]}{4\pi}}$$

$$i = \sqrt{\frac{I_m^2 [2\pi - (0)]}{4\pi}}$$

$$i_{rms} = \sqrt{\frac{I_m^2 2\pi}{4\pi}}$$

$$i_{rms} = \frac{I_m}{\sqrt{2}}$$

$$i_{rms} = 0.707 I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

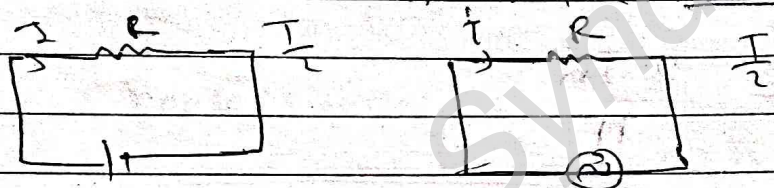


RMS value of A.C current
Analytical method.

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}}$$

Average value

The D.C current which transfer same charge in same resistance for same time as transferred by D.C. current is the average current



$$Q_{dc} = I \times \frac{T}{2}$$

$$Q_{ac} = \frac{i_1 T}{2n} + \frac{i_2 T}{2n} + \dots + \frac{i_n T}{2n}$$

$$Q_{ac} = \frac{T}{2} \left[\frac{i_1 + i_2 + i_3 + \dots + i_n}{n} \right]$$

By the definition —

$$Q_{dc} = Q_{ac}$$

$$I \frac{T}{2} = \frac{T}{2} \left[\frac{i_1 + i_2 + \dots + i_n}{n} \right]$$

$$I_{avg} = \frac{i_1 + i_2 + \dots + i_n}{n}$$

Analytical method

$$I_{avg} = \frac{1}{\pi} \int_0^{\pi} i \, d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin \theta \, d\theta$$

$$= \frac{I_m}{\pi} [-\cos \theta]_0^{\pi}$$

$$= \frac{I_m}{\pi} [1 + 1]$$

$$= \frac{2 I_m}{\pi}$$

$$I_{avg} = 0.68 I_m$$

* Form factor

It is the ratio of rms value to average value of AC current.

$$\text{Form factor} = \frac{\text{Rms value}}{\text{avg value}}$$

For sine wave $I_{rms} = \frac{I}{\sqrt{2}}$, $I_{avg} = \frac{2 I_m}{\pi}$

$$\text{Form factor} = \frac{I_m / \sqrt{2}}{2 I_m / \pi}$$

$$\text{Form Factor} = \frac{\pi}{2\sqrt{2}}$$

$$\boxed{\text{Form Factor} = 1.11}$$

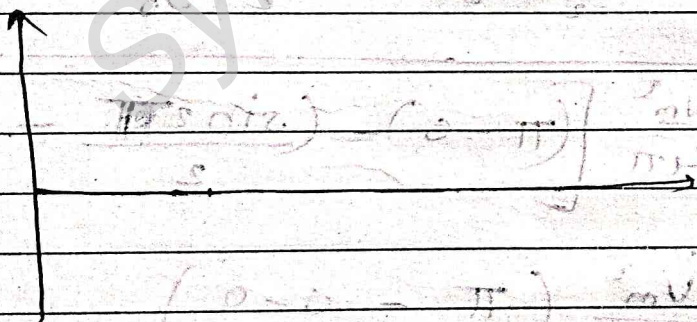
Peak Factor (crest factor)

It is a ratio of maximum (peak) value to the RMS value

For sine wave $I_{\text{max}} = I_m$, $I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$

$$\text{Peak Factor} = \frac{I_m}{I_{\text{rms}}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2}$$

$$\boxed{\text{Peak Factor} = 1.414}$$



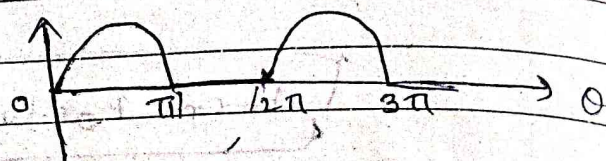
$$(x) \frac{mV}{mV}$$

$$\frac{mV}{mV}$$

Q.3 $\frac{2 \sin \omega t}{2}$

Problem (rms & average value)

1) The wave a.c.



2) Find Rms & average value of following

$$V_{rms} = \sqrt{\frac{1}{2\pi} \left[\int_0^{\pi} V_1^2 d\theta + \int_{\pi}^{2\pi} V_2^2 d\theta \right]}$$

$$= \sqrt{\frac{1}{2\pi} \left[\int_0^{\pi} V_m^2 \sin^2 \theta d\theta + \int_{\pi}^{2\pi} 0 d\theta \right]}$$

$$= \sqrt{\frac{1}{2\pi} \left[V_m^2 \int_0^{\pi} \frac{(1 - \cos 2\theta)}{2} d\theta \right]}$$

$$= \sqrt{\frac{1}{2\pi} \frac{V_m^2}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi} \left[(\pi - 0) - \left(\frac{\sin 2\pi}{2} - \frac{\sin 0}{2} \right) \right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi} (\pi - 0 - 0)}$$

$$= \sqrt{\frac{V_m^2}{4\pi} (\pi)}$$

$$= \sqrt{\frac{V_m^2}{4}}$$

$$= \frac{V_m}{2}$$

$$\boxed{V_{rms} = 0.5 V_m}$$

$$V_{ave} = \frac{1}{2\pi} \int_0^\pi V_m \sin \theta d\theta$$

$$= \frac{V_m}{2\pi} [-\cos \theta]_0^\pi$$

$$= \frac{V_m}{2\pi} [-(\cos \pi - \cos 0)]$$

$$= \frac{V_m}{2\pi} [-(-1 - 1)]$$

$$= \frac{V_m (2)}{2\pi}$$

$$= \frac{V_m}{\pi}$$

$$\boxed{V_{av} = 0.318 V_m}$$

$$\left[\cos \left(\frac{\pi}{2} \right) + \sin \left(\frac{\pi}{2} \right) \right] \pi$$

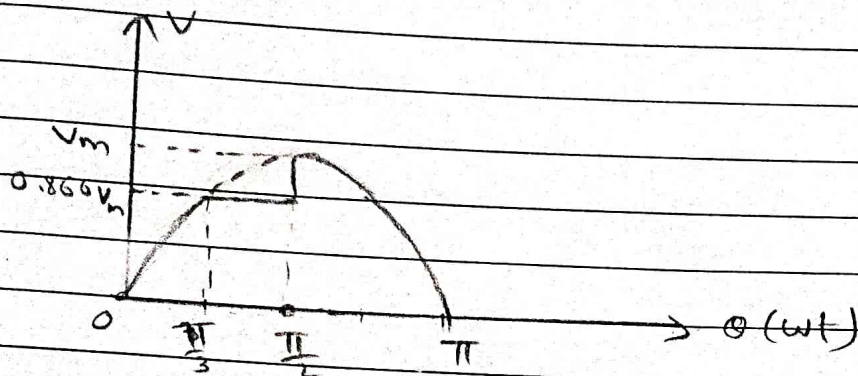
$$\frac{\pi}{\pi} \left(\cos \left(\frac{\pi}{2} \right) - 1 \right) \frac{\pi}{\pi} =$$

$$\frac{\pi}{\pi} \left(\cos \left(\frac{\pi}{2} \right) - 0 \right) \frac{\pi}{\pi} =$$

$$\frac{\pi \sin \left(\frac{\pi}{2} \right)}{\pi} \left(\frac{\pi}{\pi} - \frac{\pi}{\pi} \right) \frac{\pi}{\pi} =$$

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Q1) Find V_{AV} & V_{RMS} .



$$V = V_m \sin \theta, \quad 0 < \theta < \frac{\pi}{3}$$

$$V = 0.866 V_m, \quad \frac{\pi}{3} < \theta < \frac{\pi}{2}$$

$$V = V_m \sin \theta, \quad \frac{\pi}{2} < \theta < \pi$$

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_0^{\pi} V^2 d\theta}$$

$$V_{rms}^2 = \frac{1}{\pi} \int_0^{\pi} V^2 d\theta$$

$$= \frac{1}{\pi} \left[\int_0^{\pi/3} V^2 d\theta + \int_{\pi/3}^{\pi/2} V^2 d\theta + \int_{\pi/2}^{\pi} V^2 d\theta \right]$$

$$= \frac{1}{\pi} \left[\int_0^{\pi/3} (V_m \sin \theta)^2 d\theta + \int_{\pi/3}^{\pi/2} (0.866 V_m)^2 d\theta + \int_{\pi/2}^{\pi} (V_m \sin \theta)^2 d\theta \right]$$

$$= \frac{V_m^2}{\pi} \left[\int_0^{\pi/3} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta + \int_{\pi/3}^{\pi/2} (0.866)^2 d\theta + \int_{\pi/2}^{\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta \right]$$

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$$\begin{aligned}
 &= \frac{V_m^2}{\pi} \left[\frac{1}{2} \left(\theta - \frac{\sin 2\theta}{2} \right) \Big|_0^{\pi/3} + (0.866)^2 (\theta) \Big|_{\pi/3}^{\pi/2} + \left(\frac{1}{2} \theta - \frac{\sin 2\theta}{2} \right) \Big|_{\pi/2}^{\pi} \right] \\
 &= \frac{V_m^2}{\pi} \left\{ \frac{1}{2} \left[\frac{\pi}{3} - \frac{\sin 2(\frac{\pi}{3})}{2} - 0 + \frac{\sin 2 \times 0}{2} \right] + (0.866)^2 \left[\frac{\pi}{2} - \frac{\pi}{3} \right] \right. \\
 &\quad \left. + \frac{1}{2} \left[-\frac{\sin 2\pi}{2} + \pi - \frac{\pi}{2} + \frac{\sin 2(\frac{\pi}{2})}{2} \right] \right\} \\
 &= \frac{V_m^2}{\pi} \left[\frac{1}{2} (1.047 - 0.433) + (0.866)^2 (1.57 - 1.047) \right. \\
 &\quad \left. + \frac{1}{2} (3.14 - 0 - 1.57 - 0) \right] \\
 &= \frac{V_m^2}{3.14} (0.307 + 0.392 + 0.785)
 \end{aligned}$$

$$V_{rms}^2 = 0.4726 V_m^2$$

$$V_{rms} = 0.687 V_m$$

$V_{av} = \frac{\text{Area of full cycle}}{\text{Baselength of full cycle}}$

$$V_{av} = \frac{\int_0^{\pi} v d\theta}{\pi}$$

$$= \frac{1}{\pi} \left[\int_0^{\pi/3} v d\theta + \int_{\pi/3}^{\pi/2} v d\theta + \int_{\pi/2}^{\pi} v d\theta \right]$$

$$= \frac{1}{\pi} \left[\int_0^{\pi/3} V_m \sin \theta d\theta + \int_{\pi/3}^{\pi/2} 0.866 V_m d\theta + \int_{\pi/2}^{\pi} V_m \sin \theta d\theta \right]$$

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$$= \frac{V_m}{\pi} \left[\int_0^{\pi/3} \sin \theta d\theta + \int_{\pi/3}^{\pi/2} 0.866 d\theta + \int_{\pi/2}^{\pi} \sin \theta d\theta \right]$$

$$= \frac{V_m}{\pi} \left[(-\cos \theta)_0^{\pi/3} + 0.866 \left[\theta \right]_{\pi/3}^{\pi/2} + (-\cos \theta)_{\pi/2}^{\pi} \right]$$

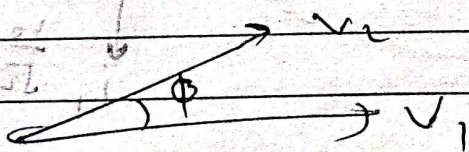
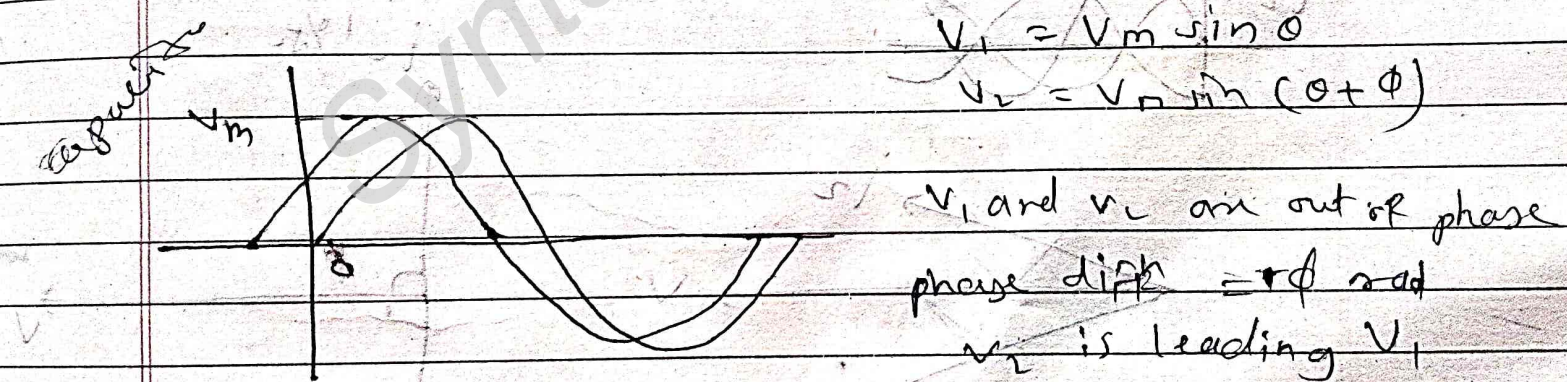
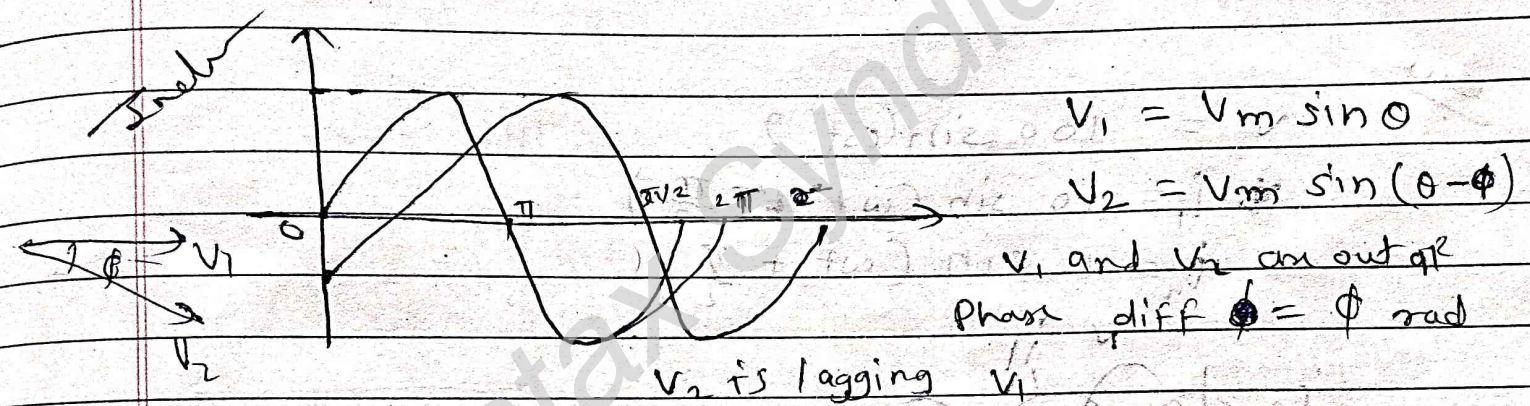
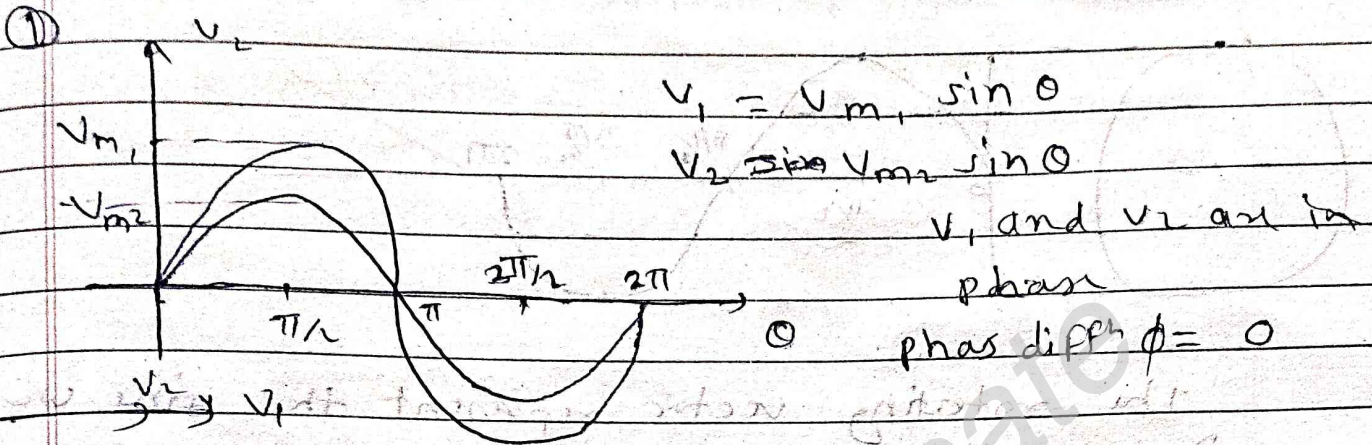
$$= \frac{V_m}{\pi} \left[\left[\cos \frac{\pi}{3} + \cos 0 \right] + 0.866 \left[\frac{\pi}{2} - \frac{\pi}{3} \right] + \left[-\cos \pi + \cos \frac{\pi}{2} \right] \right]$$

$$= \frac{V_m}{\pi} \left[\left(-\frac{1}{2} + 1 \right) + 0.866 \left(\frac{\pi}{6} \right) + 1 \right]$$

$$= \frac{V_m}{\pi} (1.953)$$

$$\boxed{V_{av} = 0.621 V_m}$$

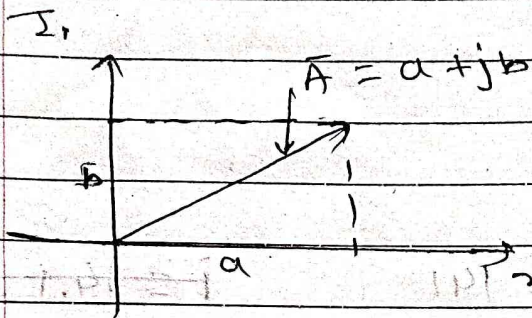
Phase difference



Phasor Algebra

① Rectangular Form

$$\bar{A} = a + jb$$



② Trigonometric Form

$$|\bar{A}| = \sqrt{a^2 + b^2}$$

$$\tan \theta = \frac{b}{a}$$

$$\theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$\sin \theta = \frac{b}{|\bar{A}|}$$

$$\cos \theta = \frac{a}{|\bar{A}|}$$

$$a = |\bar{A}| \cos \theta$$

$$b = |\bar{A}| \sin \theta$$

$$\bar{A} = a + jb$$

$$\bar{A} = |\bar{A}| \cos \theta + j|\bar{A}| \sin \theta$$

- ① Two sinusoidal current are given
 $i_1 = 10\sqrt{2} \sin(\omega t)$ & $i_2 = 10\sqrt{2} \sin(\omega t + 60)$
 Find the expression for sum of the current.

$$\rightarrow i_1 = 10\sqrt{2} \sin(\omega t) = 10\sqrt{2} \angle 0$$

$$i_2 = 10\sqrt{2} \sin(\omega t + 60) = 10\sqrt{2} \angle 60$$

$$i = i_1 + i_2 = 10\sqrt{2} \angle 0 + 10\sqrt{2} \angle 60 = 37.41 \angle 40.89$$

$$i = 37.41 \sin(\omega t + 40.89)$$

- ② Following three current are following into a junction
 $i_1 = 3\sqrt{2} \sin \omega t$, $i_2 = 5\sqrt{2} \sin(\omega t + 30)$, &
 $i_3 = 6\sqrt{2} \sin(\omega t - 120)$
 Find the expression for resultant current.

$$i_1 = 3\sqrt{2} \angle 0, i_2 = 5\sqrt{2} \angle 30, i_3 = 6\sqrt{2} \angle -120$$

$$i = i_1 + i_2 + i_3 = 7.21 \sin(\omega t - 31.9)$$

$$\boxed{i = 7.21 \sin(\omega t - 31.9)}$$

- ③ Find The resultant voltage in RMS and also expression $V_1 = 20 \sin \omega t$, $V_2 = 30 \sin(\omega t - \pi/4)$, $V_3 = 40 \cos(\omega t - \pi/6)$.

$$V_3 = 40 \sin\left(\frac{\pi}{2} + \omega t + \frac{\pi}{6}\right)$$

$$V_3 = 40 \sin(\omega t + 120)$$

$$V_1 = 20 \angle 0, V_2 = 30 \angle -45, V_3 = 40 \angle 120$$

$$V = V_1 + V_2 + V_3 = 25.1 \angle 32.32^\circ$$

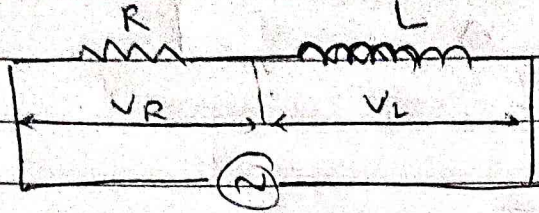
$$V = 25.1 \sin(\omega t + 32.32^\circ)$$

$$V_m = 25.1$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{25.1}{\sqrt{2}} = 17.74$$

$$V_{rms} = 17.14$$

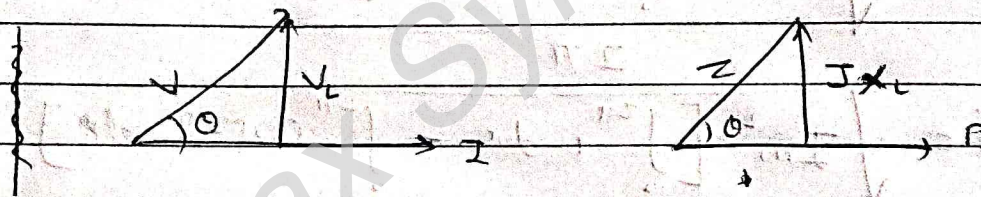
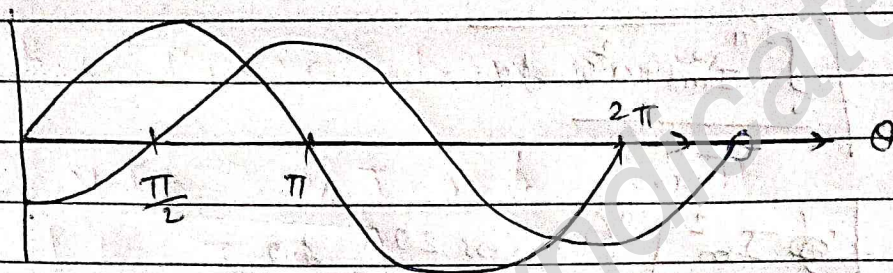
* R-L circuit



$$V = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \theta)$$

$$X_L = \omega L = 2\pi f L$$



Impedance

$$\bar{Z} = R + jX_L$$

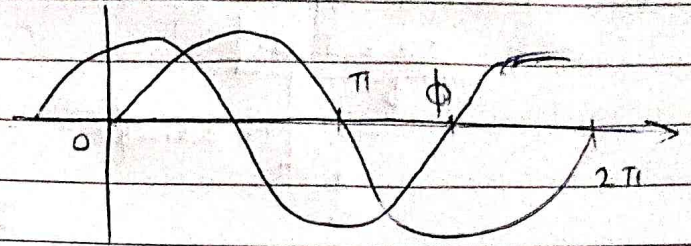
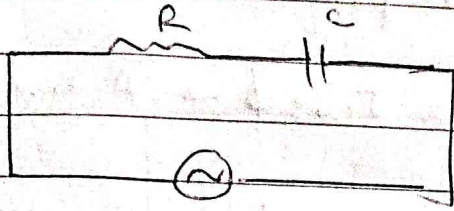
$$|\bar{Z}| = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\bar{Z} = |\bar{Z}| \angle \theta$$

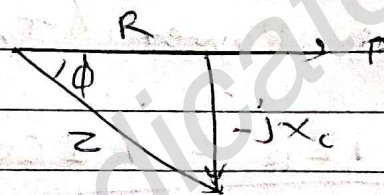
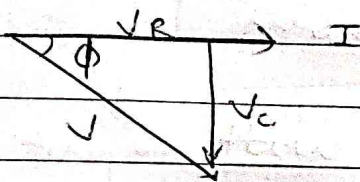
† lagging V by θ

* R-C circuit



$$V = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \phi)$$



$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\bar{Z} = R - jX_C$$

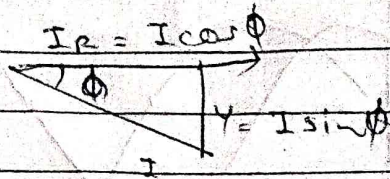
$$|\bar{Z}| = \sqrt{R^2 + X_C^2}$$

$$\theta = \tan^{-1} \left(\frac{-X_C}{R} \right)$$

$$\bar{Z} = |\bar{Z}| \angle \phi$$

i leading V by ϕ

* Active and Reactive component of current.



$I_R = I \cos \phi \rightarrow$ Active component

$I_Y = I \sin \phi \rightarrow$ Reactive component

* Power in AC circuit

① Active Power / True power

$$P = VI_R = VI \cos \phi \quad \text{watt} \leftarrow \text{unit}$$

$$\boxed{P = I^2 R}$$

② Reactive power

$$Q = VI_X = VI \sin \phi \quad \text{VAR} \leftarrow \text{unit}$$

$$\boxed{Q = I^2 X}$$

③

Apparent power

$$P = VI \cos \phi$$

$$Q = VI \sin \phi$$

$$S = P - jQ$$

$$|S| = \sqrt{P^2 + Q^2}$$

$$\boxed{S = VI} \quad \text{VA}$$

$$\boxed{S = I^2 Z} \quad \text{VA}$$

* Power Factor

- i) Power Factor of an AC circuit is define as the Ratio of Active or true power with Apparent power

$$P.F = \frac{P}{S} \quad \text{unit} \quad \frac{KW}{KVA}$$

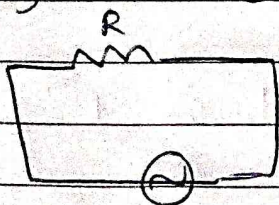
- ii) Power factor of an AC circuit is also be define as Cosine of angle betw voltage and current.

$$P.F = \cos \phi$$

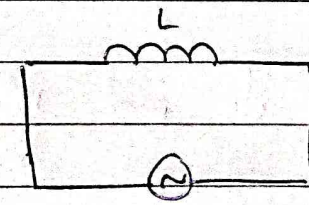
- iii) Power factor of an AC circuit can also be define as ratio of resistance to impedance of circuit.

$$\cos \phi = \frac{R}{Z} = \frac{P}{S} = P.F$$

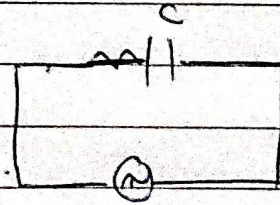
Significance of Power Factor



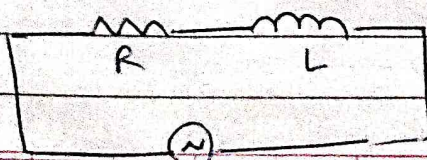
$$P.F = \cos \phi = 1$$



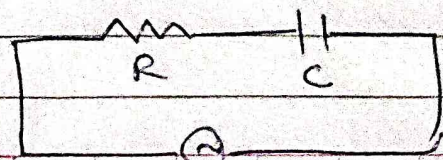
$$P.F = 0 \text{ lagging}$$



$$P.F = 0 \text{ Leading}$$

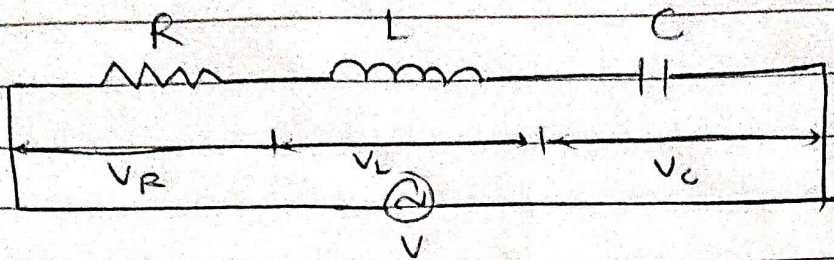


$$P.F = 1 > \cos \phi > 0 \text{ lagging}$$



$$P.F. = 1 > \cos \phi > 0 \text{ leading}$$

* R-L-C circuit.

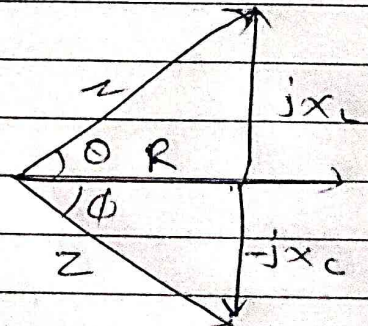
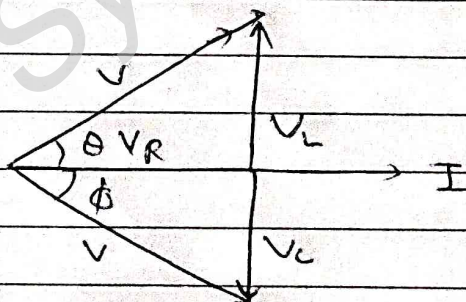
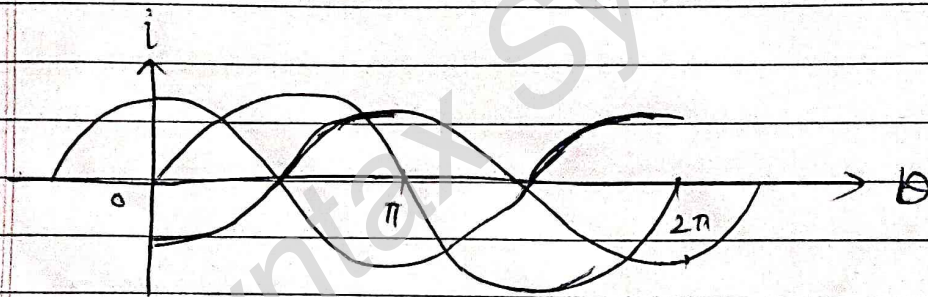


$$V = V_m \sin(\omega t)$$

$$i = I_m \sin(\omega t + \phi)$$

$$X_L = \omega L = 2\pi f L$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$



$$Z = R + j(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$Z = |\bar{Z}| \angle \phi$$

IF $X_L > X_C$: The circuit is inductive,
i lags V by angle ϕ

IF $X_C > X_L$: The circuit is capacitive
i leads V by angle ϕ

IF $X_C = X_L$: The circuit is in resonance
Then i is in phase $\phi = 0$
with voltage

Ques A voltage of $V = (150 + j180)V$ is applied across an impedance and the current is $I = (5 - j4)A$. Determine components of the circuit of power consumed.

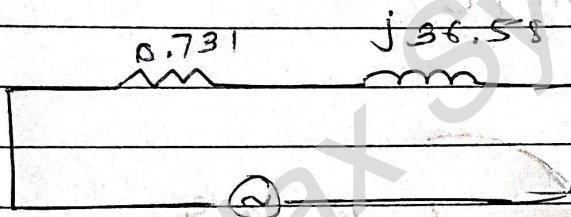
$$\begin{aligned} \rightarrow V &= (150 + j180)V = 234.3 \angle 50.19^\circ \\ I &= (5 - j4)A = 6.4 \angle -38.65^\circ \end{aligned}$$

Ohm's law

$$Z = \frac{V}{I} = \frac{150 + j180}{5 - j4}$$

$$Z = 0.731 + j36.58$$

$$Z = 36.59 \angle 88.85^\circ$$



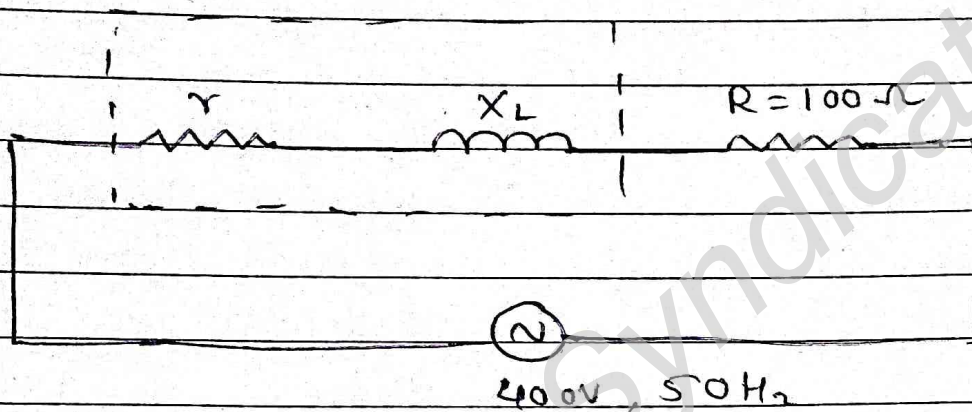
$$Z = 36.59 \quad \phi = 88.85^\circ$$

$$P = VI \cos \phi$$

$$= 234.3 \times 6.4 \times \cos(88.85^\circ)$$

$$P = 30.09 \text{ W}$$

A $100\ \Omega$ resistor is connected in series with choke coil. When a 400 V , 50 Hz is applied to this combination, the voltage across the resistance and the choke coil are 200 V and 300 V resp. Find the power consumed by the choke coil. Also calculate the power factor of the choke coil and power factor of the circuit.



Given : $R = 100\ \Omega$, $V = 400\text{ V}$, $f = 50\text{ Hz}$, $V_R = 200\text{ V}$
 $V_{\text{coil}} = 300\text{ V}$

1) power consumed by the coil

$$I = \frac{V_R}{R} = \frac{200}{100} = 2\text{ A}$$

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{300}{2} = 150\ \Omega$$

$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2}$$

$$150 = \sqrt{r^2 + X_L^2}$$

$$r^2 + X_L^2 = 22500 \quad \text{--- (1)}$$

$$Z = \frac{V}{I} = \frac{400}{2} = 200 \Omega$$

$$Z = \sqrt{(R+r)^2 + X_L^2} = 200$$

$$(100+r)^2 + X_L^2 = 40000 \quad \text{--- (2)}$$

subtract eqn (1) from (2)

$$(100+r)^2 - r^2 = 40000 - 22500$$

$$10000 + 200r + r^2 - r^2 = 17500$$

$$200r = 7500$$

$$\boxed{r = 37.5 \Omega}$$

Put r in eqn (1)

$$(37.5)^2 + X_L^2 = 22500$$

$$X_L^2 = 22500 - 1406.25$$

$$X_L^2 = 21093.75$$

$$\boxed{X_L = 145.24 \Omega}$$

$$P_{\text{coil}} = I^2 r = (2)^2 \times 37.5 = 50 \text{ W}$$

i) Power Factor of choke coil

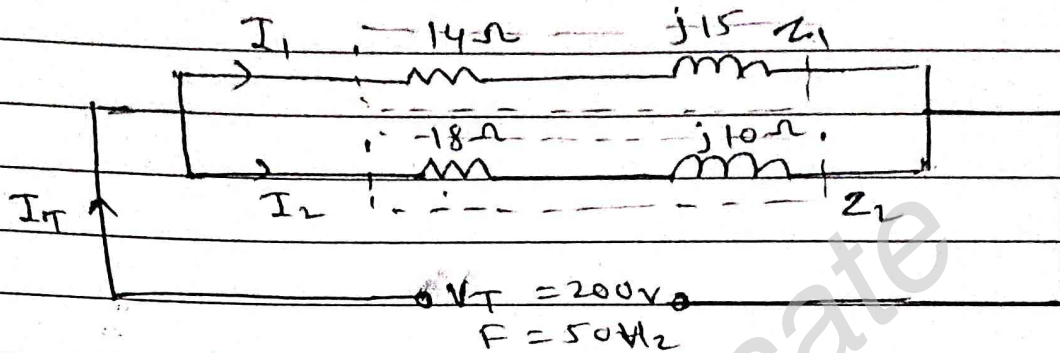
$$P.F._{\text{coil}} = \frac{r}{Z_{\text{coil}}} = \frac{37.5}{150} = 0.25$$

ii) Power Factor of the circuit

$$P.F. = \frac{R+r}{Z} = \frac{100+37.5}{200} = 0.6875$$

Two impedances $(14 + j15) \Omega$ and $(18 + j10) \Omega$ are connected in parallel across 200V, 50Hz single phase supply. Determine

- Admittance of each branch in polar form
- Current in each branch in polar form
- Power Factor of each branch
- Active power in each branch



Impedance of 1st branch

$$\bar{Z}_1 = Z_1 \angle \phi_1 \rightarrow \text{Polar Form}$$

$$\bar{Z}_1 = R_1 + jX_{L1} \rightarrow \text{Rectangular Form}$$

$$\bar{Z}_1 = 14 + j15$$

$$\bar{Z}_1 = 20.52 \angle 46.97^\circ \Omega$$

$$\boxed{Z_1 = 20.52 \Omega, \phi = 46.97^\circ}$$

Admittance of 1st branch

$$\bar{Y}_1 = \frac{1}{Z_1} = \frac{1}{14 + j15}$$

$$\bar{Y}_1 = 0.0332 - j0.0356 \text{ S}$$

$$\bar{Y}_1 = 0.0487 \angle -46.99^\circ \text{ S}$$

Impedance of 2nd branch

$$\bar{Z}_2 = R_2 + jX_{L2} = 18 + j10$$

$$\bar{Z}_2 = 20.59 \angle 29.05^\circ \Omega$$

$$\boxed{Z_2 = 20.59 \Omega, \phi = 29.05^\circ}$$

Admittance of 2nd Branch

$$\bar{Y}_2 = \frac{1}{Z_2} = \frac{1}{18 + j10}$$

$$\bar{Y}_2 = 0.042 - j0.023 \text{ } \Omega^{-1}$$

$$\bar{Y}_2 = 0.048 \angle -29.05^\circ \text{ } \Omega^{-1}$$

2) Current in each branch in Polar Form

$$I_1 = \frac{V_s}{Z_1}$$

$$I_1 = \frac{200 \angle 0^\circ}{20.52 \angle 46.77^\circ} = 9.75 \angle -46.77^\circ \text{ A}$$

$$I_2 = \frac{200 \angle 0^\circ}{20.59 \angle 29.05^\circ} = 9.72 \angle -29.05^\circ \text{ A}$$

3) Power Factor

$$\phi_1 = 46.77^\circ, \phi_2 = 29.05^\circ$$

$$\text{P.F.}_1 = \cos 46.77^\circ = 0.686$$

$$\text{P.F.}_2 = \cos 29.05^\circ = 0.874$$

4) Active power, $P_1 = VI \cos \phi_1$

$$= 200 \times 9.75 \times 0.686$$
$$[P_1 = 1336.65 \text{ W}]$$

$$P_2 = VI_2 \cos \phi_2$$

$$P_2 = 200 \times 9.72 \times 0.874$$

$$[P_2 = 1698.26 \text{ W}]$$

5) Reactive Power $Q_1 = VI_1 \sin \phi_1$

$$= 200 \times 9.75 \times \sin 46.77^\circ$$
$$[Q_1 = 1415.94 \text{ VAR}]$$

$$Q_2 = VI_2 \sin \phi_2 = 200 \times 9.72 \times \sin 29.05^\circ$$

$$[Q_2 = 944.43 \text{ VAR}]$$



~~Star and Delta~~

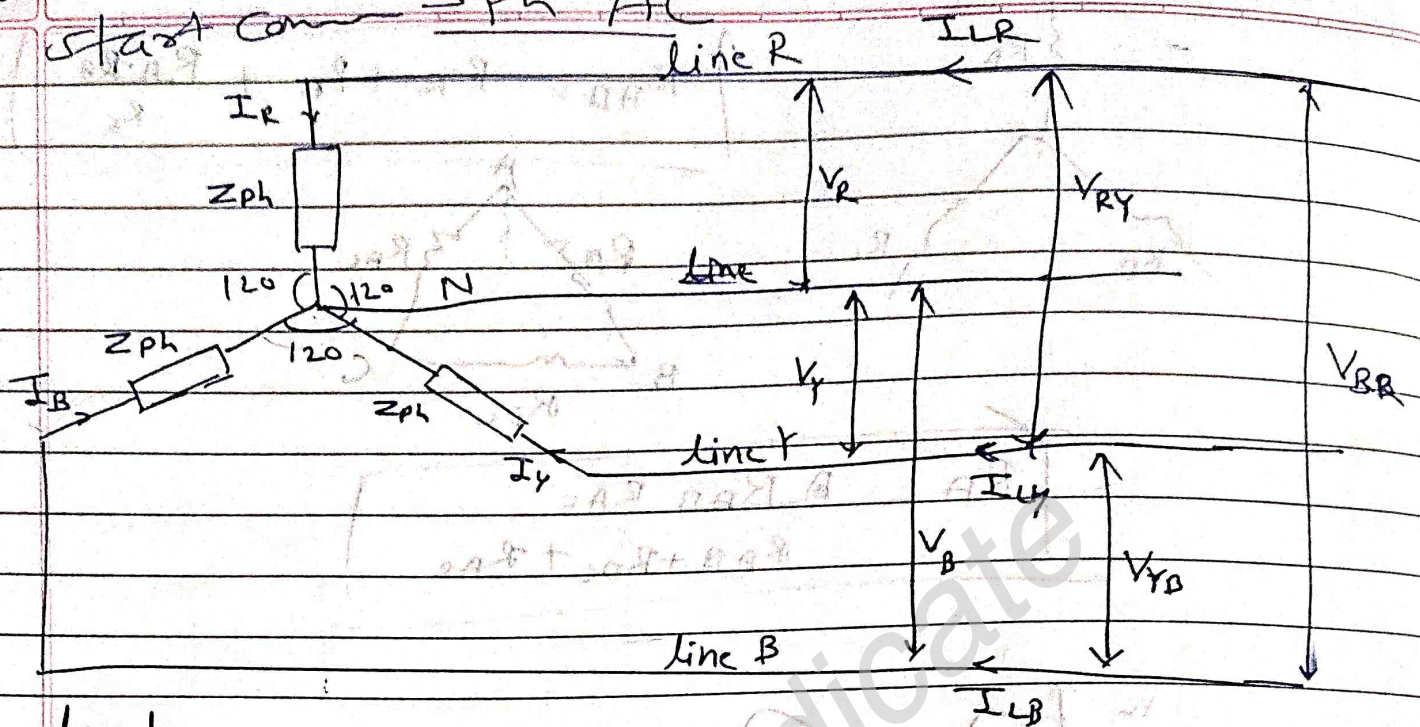
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/ /

3 Ph AC

Star Conn



load

$$I_{LR} = I_Y = I_B = I_{ph}$$

$$V_R = V_Y = V_B = V_{ph}$$

line

$$I_{LR} = I_{LY} = I_{LB} = I_L$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

① Relation betⁿ I_{ph} and I_L

From circuit diagram

$$I_{LR} = I_R \quad I_{LB} = I_B \quad I_{LY} = I_Y$$

$$\therefore I_L = I_{ph}$$

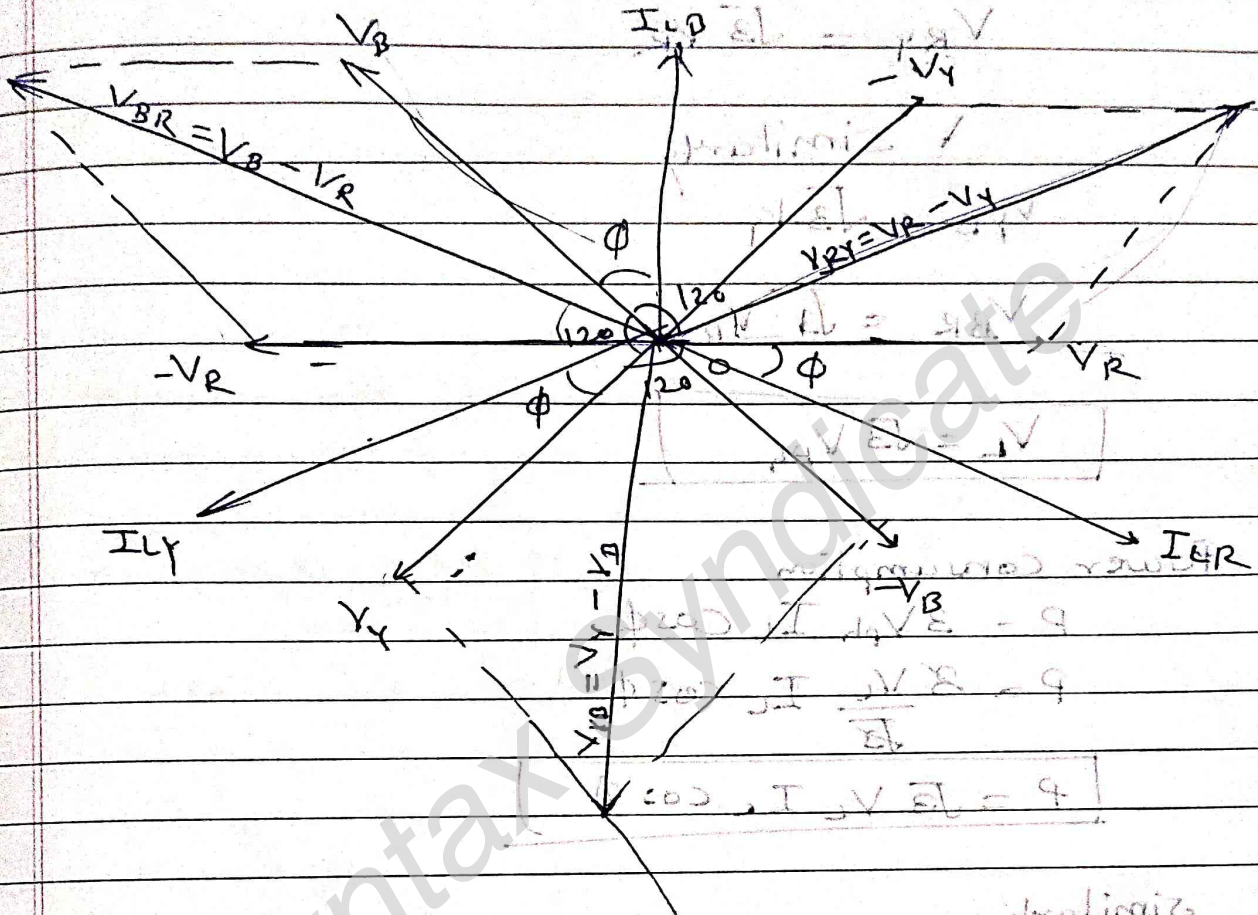
Relation between V_L and V_{ph}

$$V_{RY} = V_R - V_Y$$

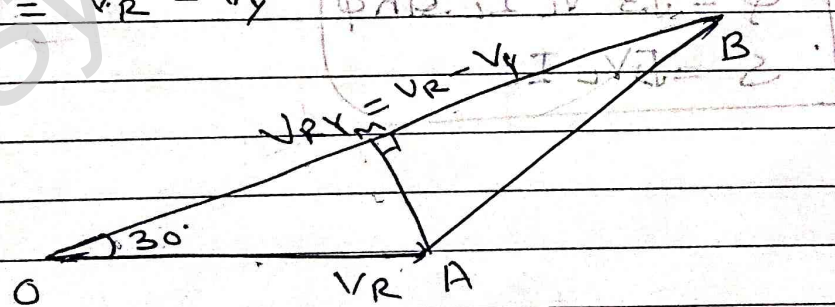
$$V_{YB} = V_Y - V_B$$

$$V_{BR} = V_B - V_R$$

Phasor diagram



$$V_{RY} = V_R - V_Y$$



$$OM = MB$$

$$OB = 2OM$$

$$OM = \frac{OB}{2}$$

$$\cos 30^\circ = \frac{OM}{OA}$$

$$OM = OA \cos 30^\circ$$

$$OB = 2OA \cos 30^\circ$$

$$V_{RY} = 2V_R \cos 30^\circ$$

$$V_{RY} = 2 V_R \frac{\sqrt{3}}{2}$$

$$V_{RY} = \sqrt{3} V_R$$

similarly

$$V_{RB} = \sqrt{3} V_R$$

$$V_{BR} = \sqrt{3} V_R$$

$$V_L = \sqrt{3} V_{ph}$$

Power Consumption

$$P = 3 V_{ph} I_L \cos \phi$$

$$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos \phi$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

similarly

$$Q = \sqrt{3} V_L I_L \sin \phi$$

$$S = \sqrt{3} V_L I_L$$

$$OM = MB$$

$$OM = MB$$

$$OM = MB$$

$$MO = 0.866$$

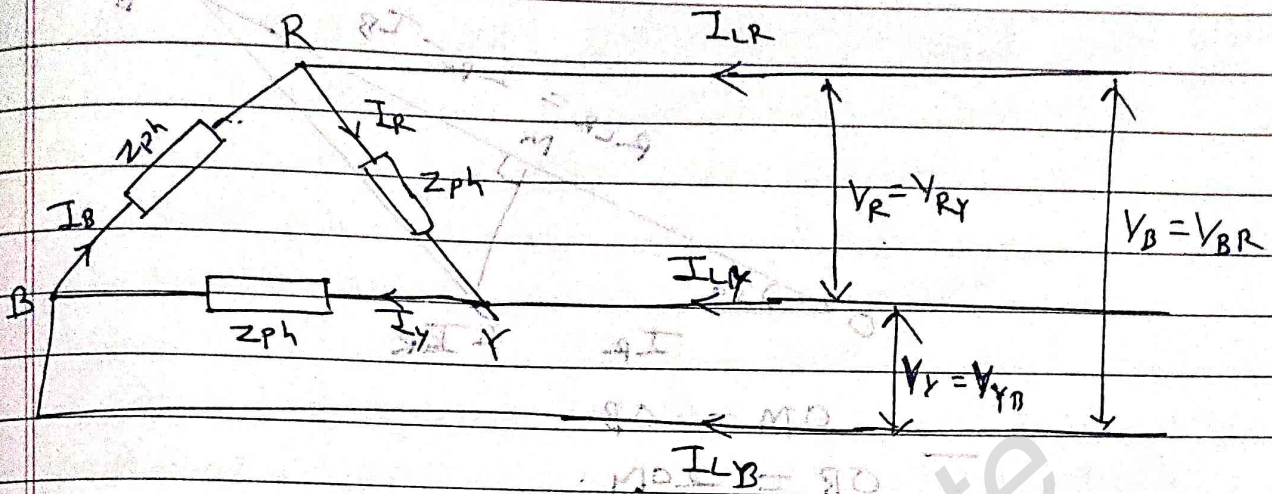
$$AO$$

$$OM = 0.866 AO$$

$$OB = 0.866 AO$$

$$OB = 0.866 AO$$

delta connection



Relation betⁿ V_L and V_{ph}

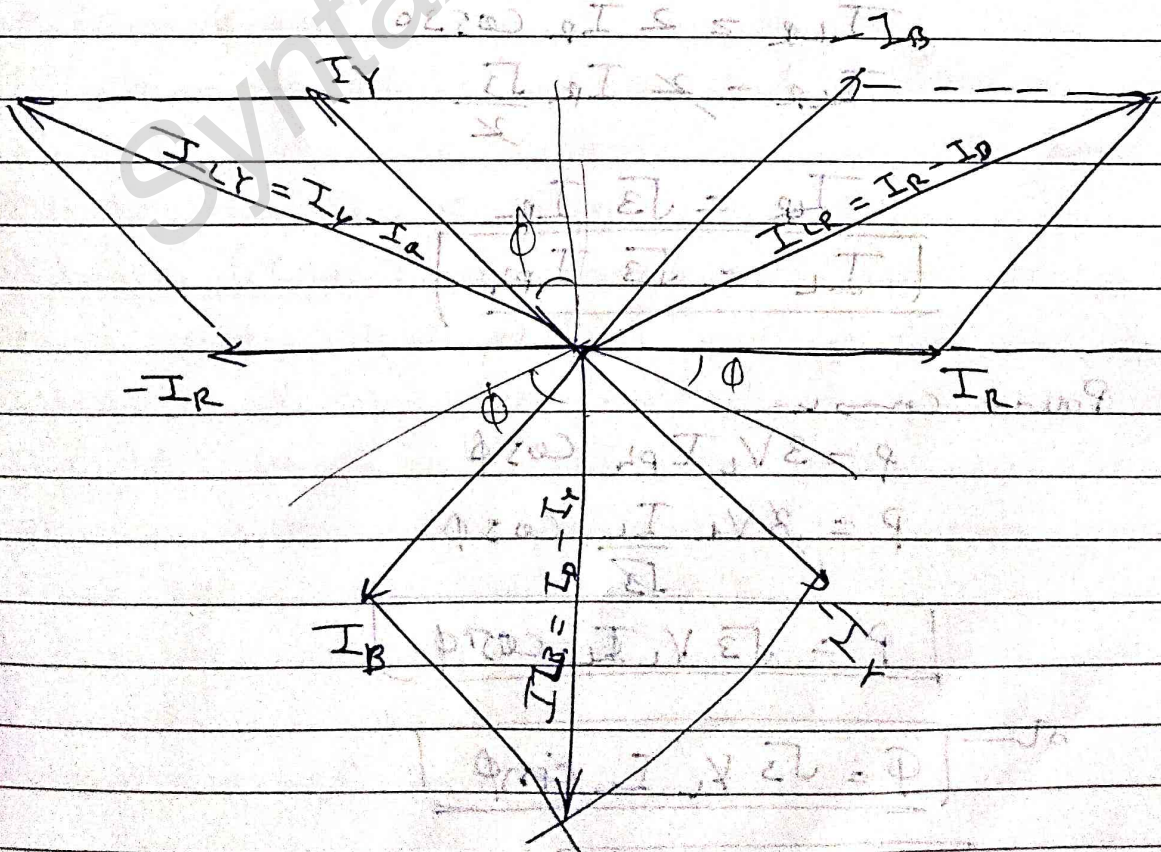
$$V_R = V_{RY}, \quad V_B = V_{BR}, \quad V_Y = V_{YB}$$

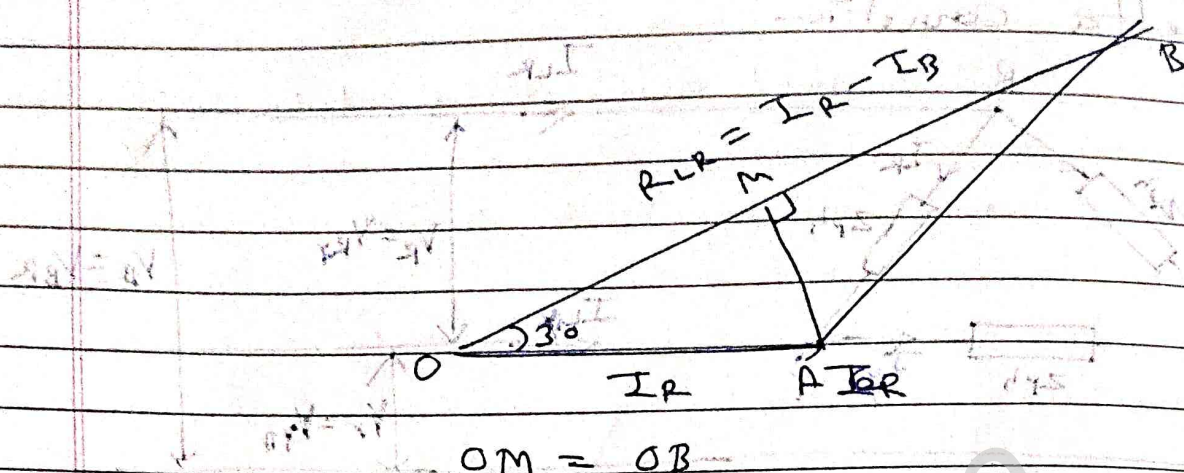
$$\therefore V_L = V_{ph}$$

$$I_{LR} = I_R - I_B$$

$$I_{LY} = I_Y - I_R$$

$$I_{LB} = I_B - I_Y$$





$$OM = OB \cos 30^\circ$$

$$OB = \frac{OM}{\cos 30^\circ}$$

$$OM = \frac{OB}{2}$$

$$\cos 30^\circ = \frac{OM}{OA}$$

$$OA = \frac{OM}{\cos 30^\circ}$$

$$OM = OA \cos 30^\circ$$

$$OB = \frac{OA \cos 30^\circ}{\cos 30^\circ} = OA = I_R$$

$$I_R = I_R \cos 30^\circ + I_B \sin 30^\circ$$

$$I_R = I_R \cos 30^\circ + I_B \sin 30^\circ$$

$$I_{LR} = 2 I_R \cos 30^\circ$$

$$I_{LR} = \frac{2}{\sqrt{3}} I_R$$

$$I_{LR} = \sqrt{3} I_R$$

$$I_L = \sqrt{3} I_{ph}$$

Power consumed

$$P = 3 V_L I_{ph} \cos \phi$$

$$P = \frac{3 V_L I_L}{\sqrt{3}} \cos \phi$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$Q = \sqrt{3} V_L I_L \sin \phi$$

$$S = \sqrt{3} V_L I_L$$

A balanced three phase load consists of 3 coils, each of resistance $4\ \Omega$ and inductance of 0.02H . It is connected to 440V , 50Hz 3 phase supply. Find the total power consumed when the load is connected in star and the total reactive power when the load is connected in delta.

$$R = 4\ \Omega, L = 0.02\text{H}, V_L = 440\text{V}, f = 50\text{Hz}$$

For a star connected load

i) Total Power Consumed

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$\bar{Z}_{ph} = R + jX_L = 4 + j6.28 = 7.45 \angle 57.51^\circ\ \Omega$$

$$Z_{ph} = 7.45\ \Omega, \phi = 57.51^\circ$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03\text{V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{7.45} = 34.1\text{A}$$

$$\boxed{I_L = I_{ph} = 34.1\text{A}}$$

$$P = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 440 \times 34.1 \times \cos(57.51^\circ)$$

$$\boxed{P = 13.96\text{ kW}}$$

$$Q = \sqrt{3} V_L I_L \sin \phi = \sqrt{3} \times 440 \times 34.1 \times \sin 57.51^\circ$$

$$\boxed{Q = 21.92\text{ KVAR}}$$

ii) When the load is connected in delta across same supply

$$Q_\Delta = 3Q_Y = 3 \times 21.92 \times 10^3$$

$$\boxed{Q_\Delta = 65.76\text{ KVAR}}$$

A balanced three phase load connected in delta draws a power of 10kW at 440V at a power factor of 0.6 lead. Find the values of circuit elements and reactive voltamperes.

$P = 10\text{ kW}$, $V_L = 440\text{ V}$, $\text{P.F.} = 0.6$ leading.
For a delta-connected elements.

i) Values of circuit elements

$$V_L = V_{ph} = 440\text{ V}$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$10 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.6$$

$$\boxed{I_L = 21.87\text{ A}}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{21.87}{\sqrt{3}}$$

$$\boxed{I_{ph} = 12.63\text{ A}}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{440}{12.63} =$$

$$\boxed{Z_{ph} = 34.84\ \Omega}$$

$$\phi = \cos^{-1}(0.6) = 53.13^\circ$$

$$R_{ph} = Z_{ph} \cos \phi = 34.84 \times 0.6$$

$$\boxed{R_{ph} = 20.90\ \Omega}$$

$$X_{ph} = Z_{ph} \sin \phi = 34.84 \times 0.8$$

$$\boxed{X_{ph} = 27.87\ \Omega}$$

ii) Reactive volt-ampere drawn

$$Q = \sqrt{3} V_L I_L \sin \phi = \sqrt{3} \times 440 \times 21.87 \times 0.8$$

$$\boxed{Q = 13.33\text{ kVAR}}$$