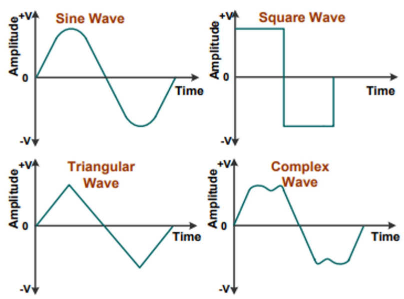
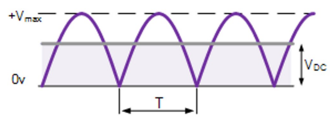
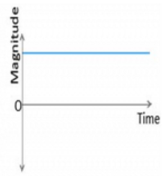
A. C. FUNDAMENTALS

08 March 2021 11:14

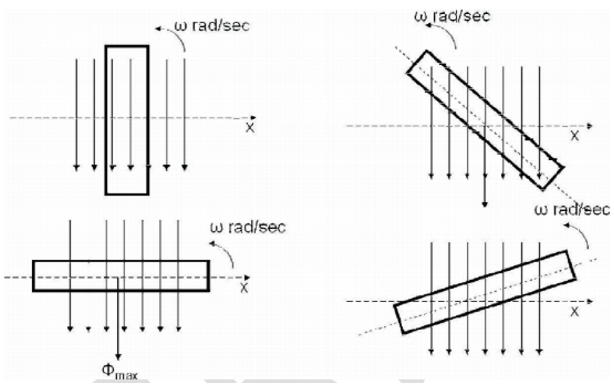
AC funda1

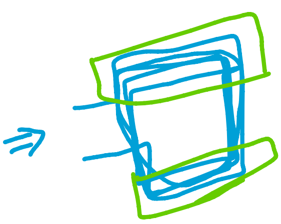
Comparison of AC and DC

Parameter of Comparison	AC Current	DC Current
Definition	An Alternating Current reverses the direction of the flow while flowing in the electric circuit.	The Direct Current is unidirectional with the flow inside the electrical circuit.
Variants	Square, Triangular, Triangular, Sinusoidal Wave form etc. 	This is pulsating in nature and pure by form. Pulsating DC   Pure DC
Source of Generation	All the Alternating Current Generators are the sources of AC currents.	All the cells (Chemical and Solar) or batteries and DC generators are the sources for Direct Current generation.
Electron Movement and Direction	The electrons keep on changing the direction of flow i.e. forward and backward direction.	The electron movement is unidirectional i.e. the flow is only in the forward direction.
Current Magnitude	The magnitude of Alternating current varies with time.	The magnitude of Direct Current does not vary with time.
Power Factor	The Power factor for AC fluctuates between 0 and 1.	The Power factor value for direct current is always 1.
Type of Load	Resistive, inductive, capacitive	Resistive only
Applications	Wide range of applications: Home, Office, Commercial, Agricultural, Industrial applications, Electric traction etc	Limited Applications: Electronic gadgets, Electric vehicles, Landline Telephones, Electrorefining and electroplating
Use of transformer	Transformer is used to step up and step down the voltage	Transformer can not be used. Therefore stepping up and step down the voltage is difficult

Generation of AC voltage: (https://www.youtube.com/watch?v=gQyamjPrw-U)

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of ω rad/sec.





When the coil is in the vertical position, the flux linking the coil is maximum but the rate of cutting the fluX is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero

When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil according to Faradays Law.

When the coil reaches the horizontal position, the flux linking the coil is zero but the rate cutting th flux is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces.

Next when the coil comes to the vertical position, the emf induced becomes zero.

After that the same cycle repeats and the emf is induced in the opposite direction.

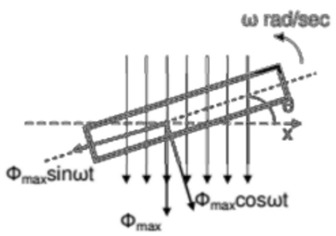
When the coil completes one complete revolution, one cycle of AC voltage is generated.

The generation of sinusoidal AC voltage can also be explained using mathematical equations.

Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure.

The maximum flux linking the coil is in the downward direction as shown in the figure.

This flux can be divided into two components, one component acting along the plane of the coil Φmaxsinωt and another component acting perpendicular to the plane of the coil Φmaxcosωt



The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil ie Φmaxcosωt induces an emf in the coil.

$$\emptyset = \emptyset_{max}\cos(\omega t)$$
$$e = -N\frac{d\emptyset}{dt}$$
$$e = -N\frac{d(\emptyset_{max}\cos(\omega t))}{dt}$$
$$e = N\emptyset_{max}\omega * \sin(\omega t)$$
$$e = E_{max}\sin(\omega t)$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$i = i_m sin\,(\omega t)$$

Where ,

i is the instantaneous value

i_m is maximum value

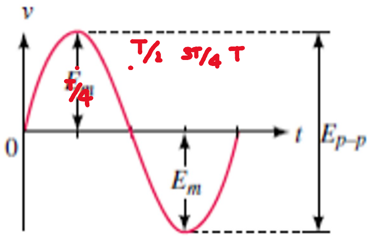
ω is angular frequency

Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil).

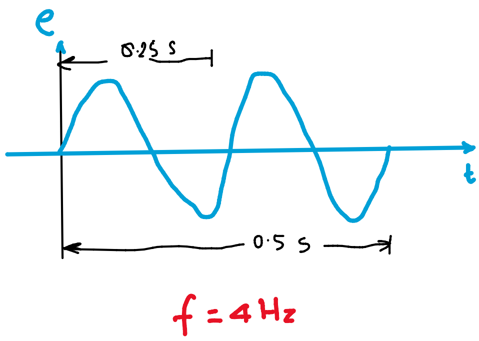
The unit of angular frequency is rad/sec.

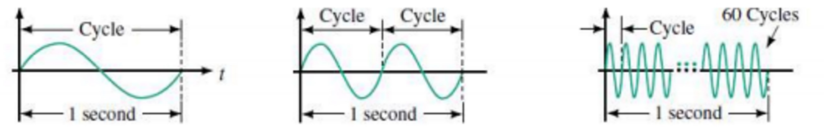
AC waveform and Definitions

Waveform: It is a graph showing the manner in which an alternating quantity changes with time.



Frequency: The number of cycles per second of a waveform is defined as its frequency. 1 Hz =1 cycle per second





(a) 1 cycle per second = 1 Hz (b) 2 cycles per second = 2 Hz (c) 60 cycles per second = 60 Hz

Period

The period (or the time period), T, of a waveform, is the duration of one cycle i.e. Time taken to complete one cycle.

It is the inverse of frequency.

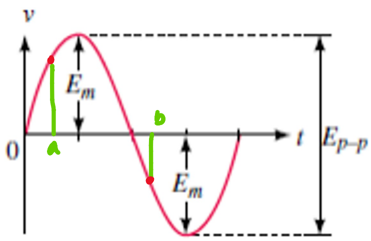
Relation between Period and frequency:

Period and frequency are inverse of each other i.e.

$$T= 1/f \quad \text{or } f = 1/T$$

Amplitude and Peak-to-Peak Value:

The amplitude of a sine wave is its positive or negative maximum value. Thus, the amplitude of the voltage in Figure below is Em

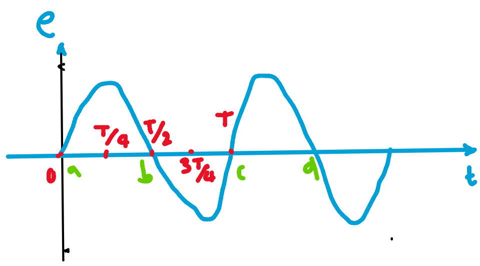


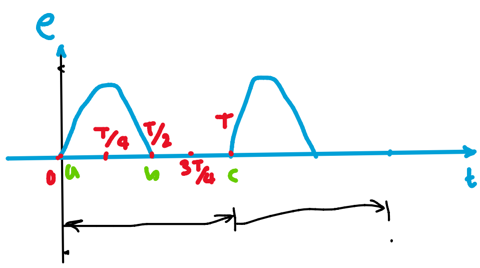
Peak-to-peak voltage is also indicated in Figure . Denoted by Ep-p or Vp-p, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

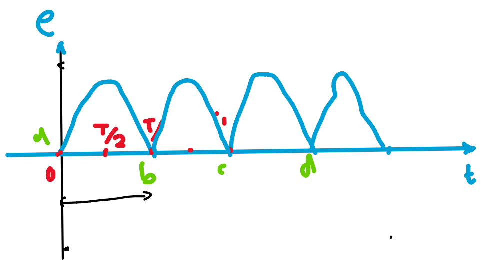
Peak Value: The peak value of a voltage or current is its maximum value with respect to zero.

Instantaneous values are the values of the alternating quantities at any instant of time. They are represented by small letters, *i, v, e*, etc.

Cycle: It is the complete set of positive and negative values of the alternating quantity.







Example

1. An ac voltage is given by $v = 100 \sin(314 \cdot t)$
   Determine Angular frequency, frequency, Time period and amplitude
   Also find the instantaneous value of the voltage at t=7.5 mS and 12.5 mS

   Soln:
   Given equation $v = 100 \sin(314 \cdot t)$

Comparing with V= Vm. Sin (ω.t)

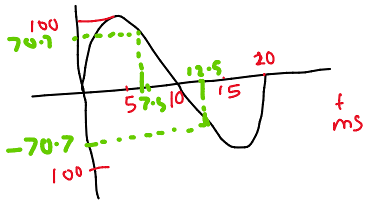
Vm=100V =Amplitude

$\omega = 314$ rad/s

$\omega = 2\pi f$

$f = \frac{314}{2\pi} = 50$ Hz

$T = \frac{1}{f} = 0.02S = 20$ mS



At t= 7.5 mS
V=70.7V

At t= 12.5 mS
V= -70.7V

QUESTION AC FUNDAMENTALS

Q1)

An alternating current takes 3.375 ms to reach 15 A for the first time after becoming instantaneously zero. The frequency of the current is 40 Hz. Find the maximum value of the alternating current. **[May 2014]**

Q2)

An alternating current of 50 c/s frequency has a maximum value of 100 A. (i) Calculate its value $\frac{1}{600}$ second and after the instant the current is zero. (ii) In how many seconds after the zero value will the current attain the value of 86.6 A?

Q3)

An alternating current varying sinusoidally with a frequency of 50 c/s has an rms value of 20 A. Write down the equation for the instantaneous value and find this value at (i) 0.0025 s, and (ii) 0.0125 s after passing through zero and increasing positively. (iii) At what time, measured from zero, will the value of the instantaneous current be 14.14 A?

Q4)

A sinusoidal wave of 50 Hz frequency has its maximum value of 9.2 A. What will be its value at (i) 0.002 s after the wave passes through zero in the positive direction, and (ii) 0.0045 s after the wave passes through the positive maximum.

Q5)

An alternating voltage is represented by v = 141.4 sin 377t. Find (i) max-value (ii) frequency (iii) time period. **[May 2016]**

Q6)

An alternating current varying sinusoidally at 50 Hz has its rms value of 10 A. Write down an equation for the instantaneous value of the current. Find the value of the current at (i) 0.0025 second after passing through the positive maximum value, and (ii) 0.0075 second after passing through zero value and increasing negatively.

Q7)

A non-sinusoidal voltage has a form factor of 1.2 and peak factor of 1.5. If the average value of the voltage is 10 V, calculate (i) rms value, and (ii) maximum value.

Q8)

Find the following parameters of a voltage v = 200 sin 314 t:
(i) frequency, (ii) form factor, and (iii) crest factor.

Example2

An alternating current takes 3.375 ms to reach 15 A for the first time after becoming instantaneously zero. The frequency of the current is 40 Hz. Find the maximum value of the alternating current. **[May 2014]**

Soln:

Given t=3.375 mS =3.375x10^{-3} S,

i =15A,

f=40 Hz

Im=?

i=Imsin(w.t)

Im= i/sin(w.t)

W=2xπxf=2xπx40=251.2 rad/s

Im=15/sin(251.2x3.375x10^{-3})

Im=20A

HW

A sinusoidal wave of 50 Hz frequency has its maximum value of 9.2 A. What will be its value at (i) 0.002 s after the wave passes through zero in the positive direction, and (ii) 0.0045 s after the wave passes through the positive maximum.

RMS and Average Values

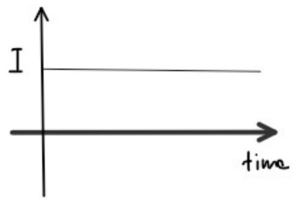
15 March 2021 05:19

RMS Value

"The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time."

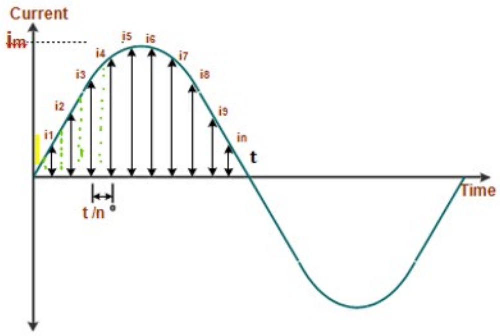
Graphical method(Mid Ordinate Method):

Hdc



$$H_{dc} = \frac{I^2 R t}{J}$$

Hac



$$H_{ac} = \frac{i_1^2 R \frac{t}{n}}{J} + \frac{i_2^2 R \frac{t}{n}}{J} + \cdots + \frac{i_n^2 R \frac{t}{n}}{J}$$

For DC current I to be the RMS value of the given AC current,

Hdc = Hac

$$\frac{I^2 R t}{J} = \frac{R \cdot t}{J}\left(\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}\right)$$

Therefore

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}.$$

According to the definition this is the RMS value of given AC current.

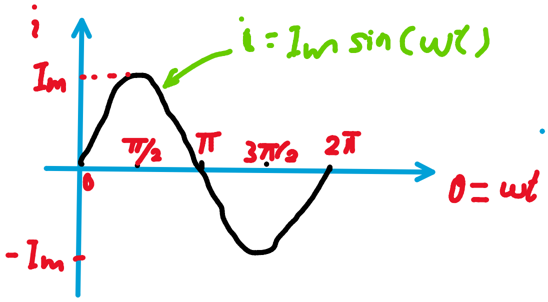
Therefore

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}$$

Similarly

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_n^2}{n}}$$

RMS Value - Analytical Method



$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t))}{2}\, d(\omega t)}$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707 i_m$$

Irms = Im/$\sqrt{2}$ = 0.707Im Similarly, Vrms = Vm/$\sqrt{2}$=0.707Vm

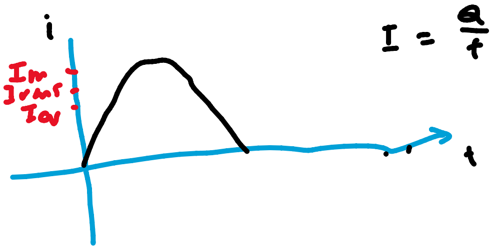
Average Value (dc Value):

"The average value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time transfers the same amount of charge as transferred by the alternating current flowing through the same resistance for the same time."

Graphical method (Mid-ordinate method)

Derivation-HW

$$I_{av} = \frac{i_1 + i_2 + \cdots + i_n}{n}$$



$I = \frac{Q}{t}$ $Q = I \cdot t$

$Q_{ac} = Q_{dc}$

Analytical method

$$I_{av} = \frac{2 I_m}{\pi} = 0.637 I_m$$

Examples

1. An ac current is given by $i = 28.28 \sin(628 \cdot t)$
   Find frequency, time period, RMS value and average value
   Soln:
   Comparing with $i = I_m \sin(\omega \cdot t)$

   $I_m = 28.28\ A$ $\omega = 628\ rad/s$

   Therefore
   F=100Hz
   T=0.01S
   Irms=20A
   Iav=18A

   Definitions:
   Form Factor: It is the ratio of RMS value to average value of the AC quantity.

$$Form\ Factor = \frac{RMS\ value}{Average\ value}$$

For sine waveform

RMS value = Im/$\sqrt{2}$

And Average value = 2.Im/π

Therefore

$$Form\ Factor = \frac{Im/\sqrt{2}}{2Im/\pi}$$

$$Form\ factor = 1.11$$

Peak Factor: It is the ratio of peak value to RMS value of the AC quantity.

$$Peak\ Factor = \frac{Peak\ value}{RMS\ value}$$

For sine waveform

RMS value = Im/$\sqrt{2}$

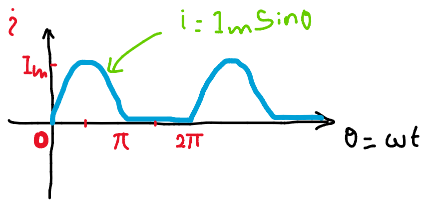
And Peak value = Im

Therefore

$$Peak\ Factor = \frac{Im}{Im/\sqrt{2}}$$

$$Peak\ Factor = \sqrt{2} = 1.414$$

Find the RMS and Average value of the following waveform



$$Irms = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I2m\ sin2\theta\ d\theta + \int_{\pi}^{2\pi} 0\ d\theta}$$

= Im/2

$$I_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin\theta \cdot d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

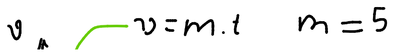
$$= \frac{I_m}{\pi}$$

Form factor = 1.57 and peak factor = 2

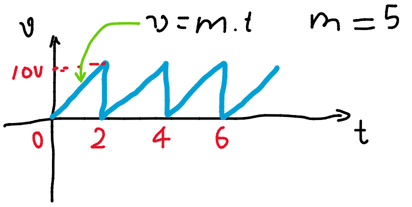
HW

Find the RMS and Average value of the full wave rectified sine waveform

Find the RMS and Average value of the following waveform



Find the RMS and Average value of the following waveform



Sol

$$v = 5 \cdot t$$

$$V_{rms} = \sqrt{\frac{1}{2}\int_0^2 v^2 dt}$$

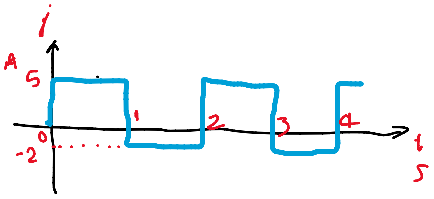
$$= \sqrt{\frac{25}{2}\int_0^2 t^2 \cdot dt}$$

$$= 5.77\,V$$

$$V_{av} = \frac{1}{2}\int_0^2 v \cdot dt$$

$$= 5\ V$$

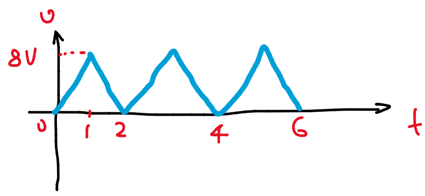
Form factor = 5.77/5=1.154
Peak factor = 10/5.77=1.73
HW
Find the RMS and Average value of the following waveform



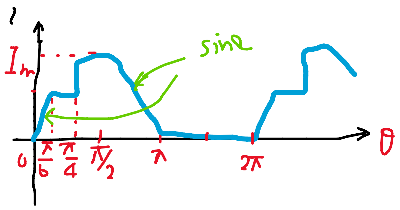
$$I_{rms} = \sqrt{\frac{1}{2}\left[\int_0^1 5^2 dt + \int_1^2 (-2)^2 dt\right]} = 3.81\,A$$

$$I_{av} = \frac{1}{2}\left[\int_0^1 5\,dt + \int_1^2 |-2|\,dt\right] = 1.5\,A$$

Find the RMS and Average value of the following waveform



Find the RMS and Average value of the following waveform



$0 - \pi/6 \qquad i = I_m \sin\theta$

$\pi/6 - \pi/4 \qquad i = 0.5 I_m$

$\pi/4 - \pi \qquad i = I_m \sin$

$\pi - 2\pi \qquad i = 0$

$\pi/4 - \pi \quad i = I_m \sin$

$\pi - 2\pi \quad i = 0$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi/6} I_m^2 \sin^2\theta \, d\theta + \int_{\pi/6}^{\pi/4} (0.5 \cdot I_m)^2 \, d\theta + \int_{\pi/4}^{\pi} I_m^2 \sin^2\theta \cdot d\theta + 0\right]}$$

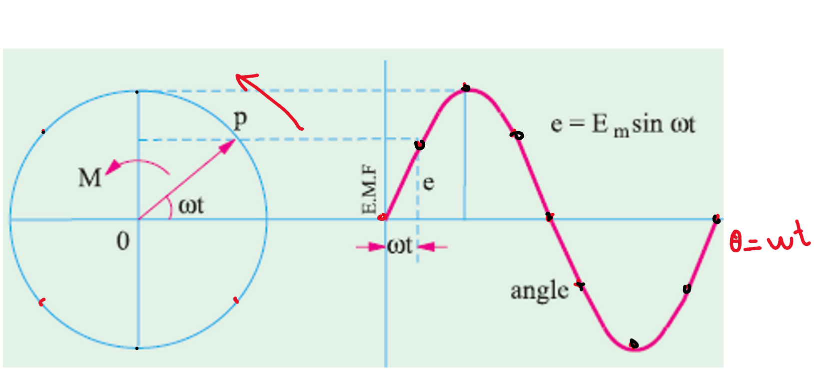
Phase and Phase Difference

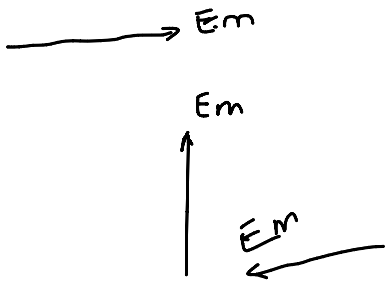
17 March 2021 04:01

Phasor (Vector) Representation

A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.

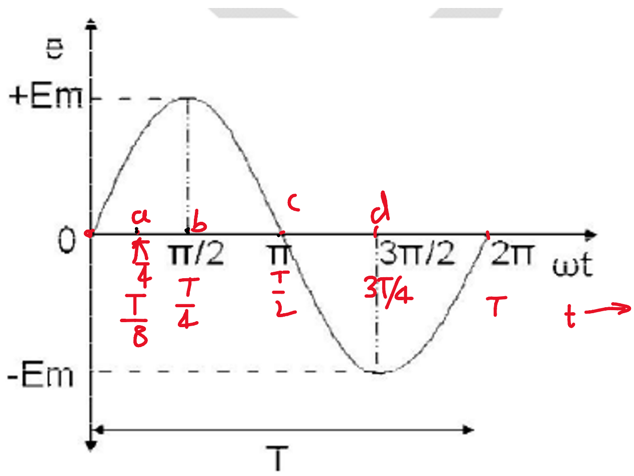




In phasor form the above wave is written as $E = E \ \angle 0^0$
Draw a line OP of length equal to E_m. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by OM=OPsinθ = $E_m$sinωt. Hence the line OP is the phasor representation of the sinusoidal current

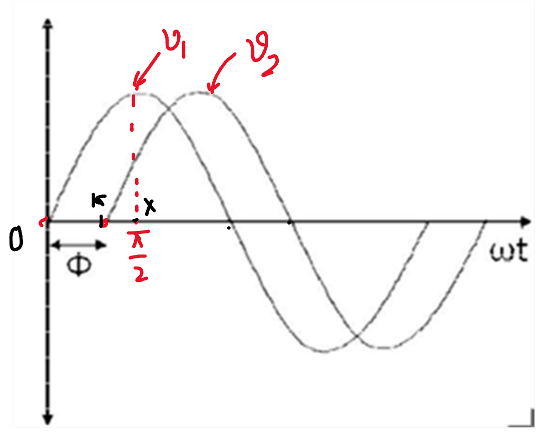
Phase:

Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference



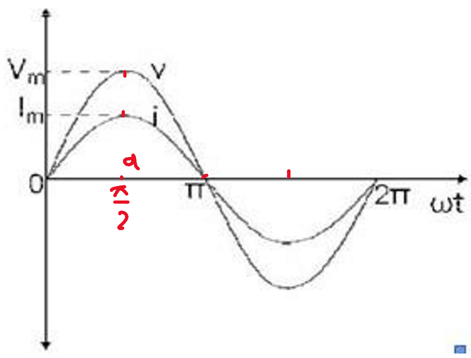
Phase Difference

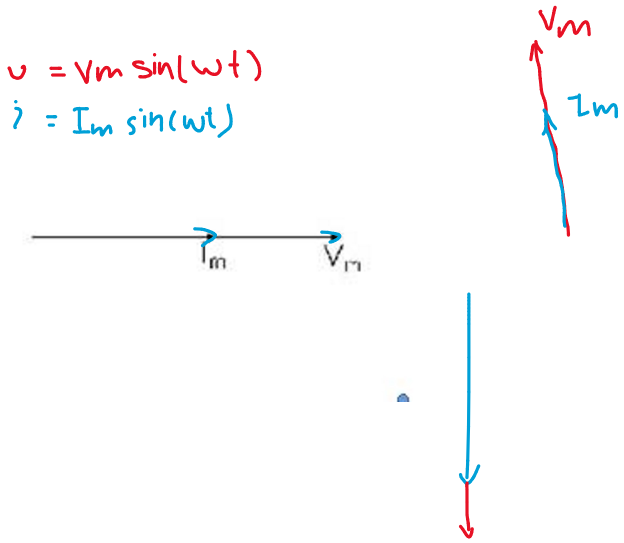
When two alternating quantities with the same frequency have different zero points, they are said to have phase difference.
The angle between the two zero points is the angle of phase difference and it is measured in radians or degrees.



In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

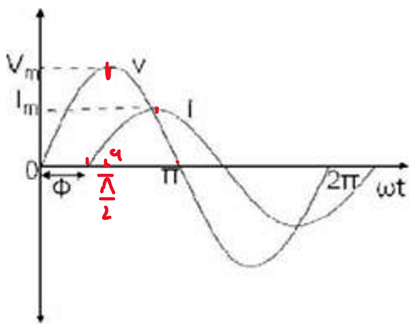


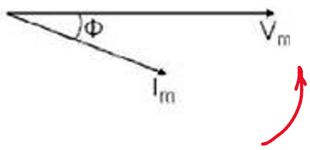


v = vm sin(ωt)
i = im sin(ωt)

Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.

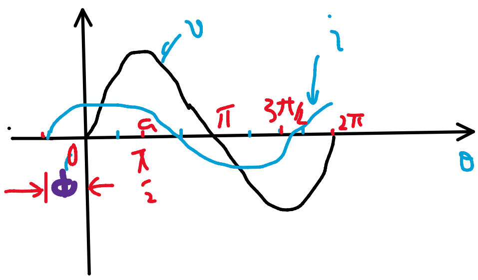




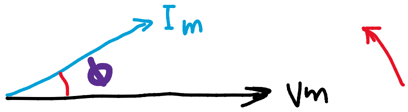
$v = vm\sin(\omega t) \Rightarrow \bar{V} = V \ \angle 0^0$
$i = im\sin(\omega t - \text{phi}) \Rightarrow \bar{I} = I \ \angle -\text{phi}$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.



$v = vm\sin(\omega t) \Rightarrow \bar{V} = V \ \angle 0^0$
$i = im\sin(\omega t + \text{phi}) \Rightarrow \bar{I} = I \ \angle \text{phi}^0$



Example

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

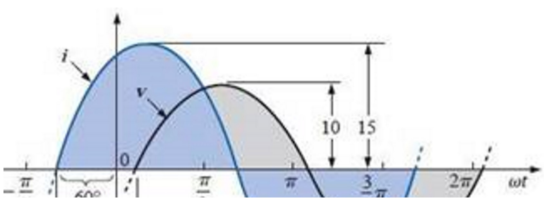
1) $i = 15\sin(wt + 60)$, $v = 10\sin(wt - 20)$

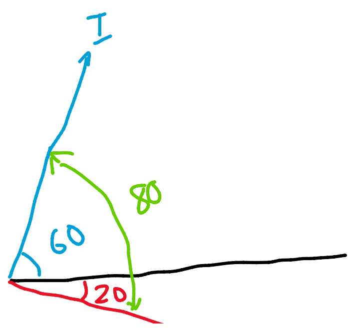
2) $i = 2\cos(wt + 10)$, $v = 3\sin(wt - 10)$

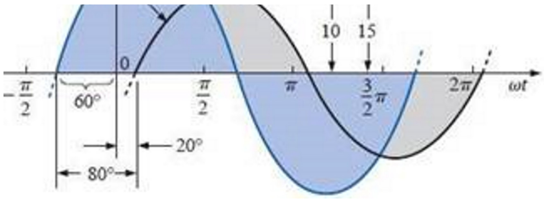
Solution:

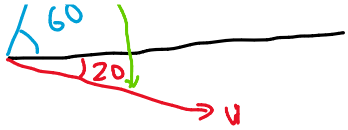
1) $i = 15\sin(wt + 60)$, $v = 10\sin(wt - 20)$

i leads v by 80°, or v lags i by 80°.









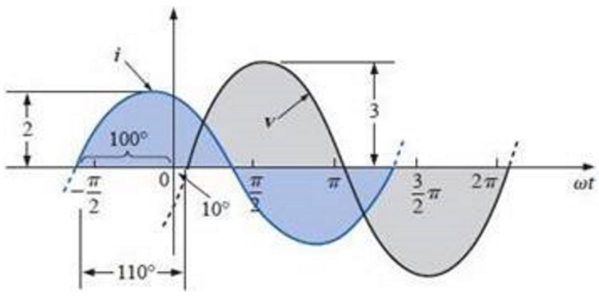
2) i = 2cos(wt +10)

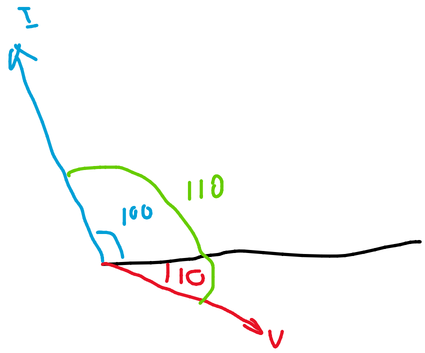
v = 3sin(wt -10)

i = 2cos(wt +10) = 2sin(wt +10+ 90) = 2sin(wt +100)

v = 3sin(wt -10)

***i* leads *v* by 110°, or *v* lags *i* by 110°.**





Phasor Algebra

Refer the attached file

AC funda1

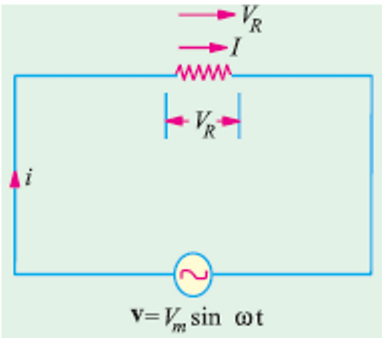
Conversion of polar and rectangular form

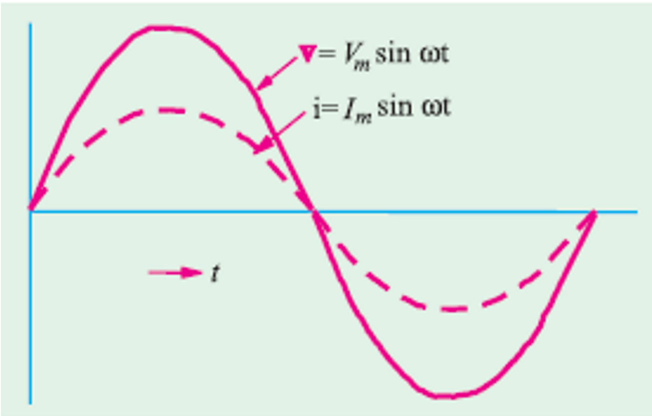
https://www.youtube.com/watch?v=K7yfOc3npJE

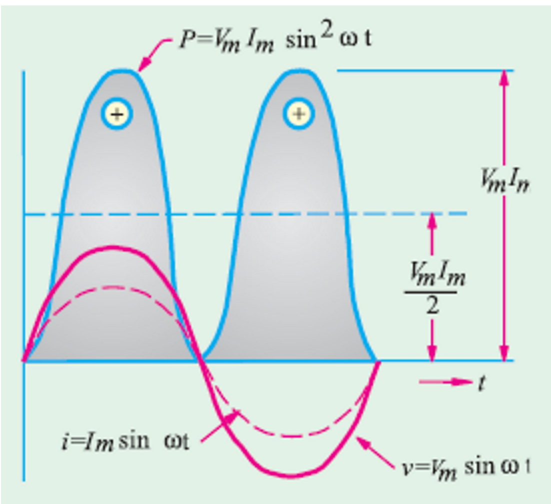
Retangular and Polar Form Conversion Casio Fx 991ES Plus

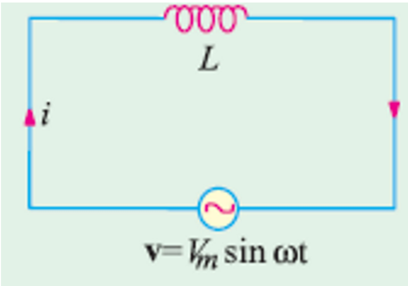
Conversion of polar to cartesian and vice versa by using a calculator

18 March 2021 09:57

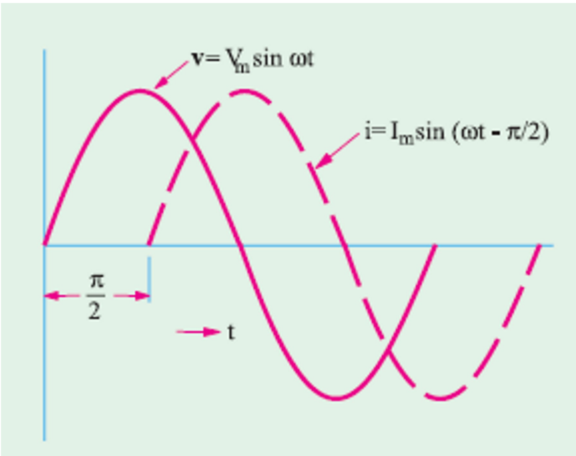




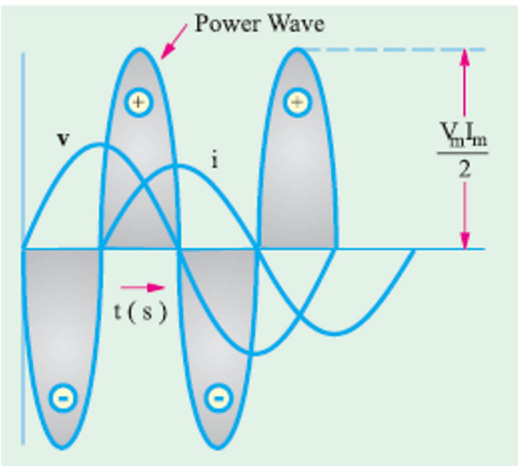




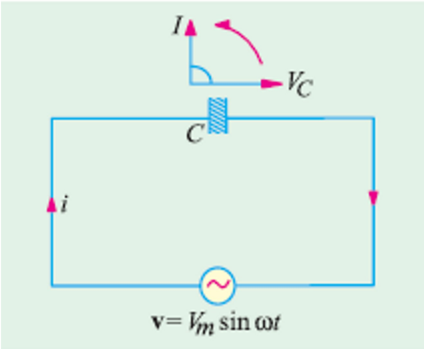
L
i
v= Vm sin ωt

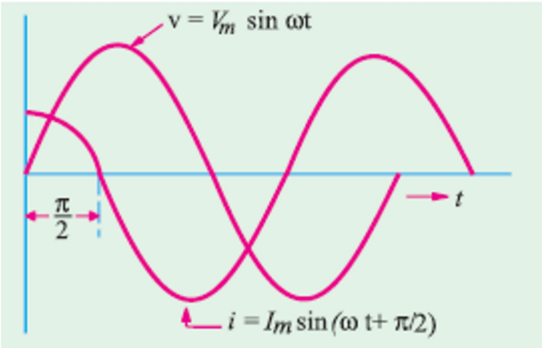


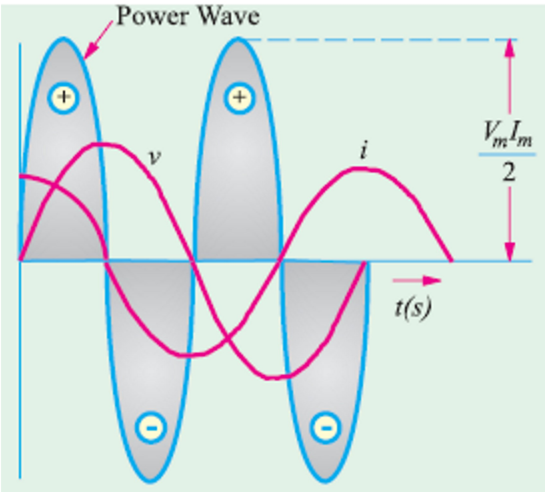
v= Vm sin ωt
i= Im sin (ωt - π/2)
π/2
t



Power Wave
v
i
VmIm/2
t (s)







A 60-Hz voltage of 115 V (r.m.s.) is impressed on a 100 ohm resistance :
(i) Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at t = 0 (ii) Show the voltage and current on a time diagram. (iii) Show the voltage and current on a phasor diagram Find the RMS value of current and power consumed

A 50-Hz voltage of 230 V (r.m.s.) is impressed on a 0.1 H inductance :
(i) Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at t = 0 (ii) Show the voltage and current on a time diagram. (iii) Show the voltage and current on a phasor diagram Find the RMS value of current and power consumed

A 60-Hz voltage of 115 V (r.m.s.) is impressed on a 100 microFarad capacitor :
(i) Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at t = 0 (ii) Show the voltage and current on a time diagram. (iii) Show the voltage and current on a phasor diagram. Find the RMS value of current and power consumed

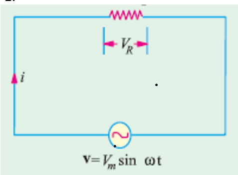
Voltage and current in a single element ac circuit are given by

Identify the circuit element and find its value

AC applied to Resistance, Inductance and Capacitance

17 March 2021 11:22

AC applied to Resistance only

1. 

2. Voltage - $v = V_m \sin(\omega t)$

3. Current -

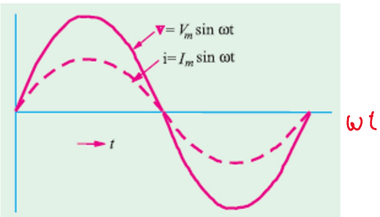
v = i.R
i = v/R
$i = I_m \sin(\omega t)$

4. Phasor Relation between V and I:

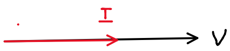
It is seen from equations that voltage and current are in-phase and phase difference (phi) is zero

Current is in phase with voltage

5. Waveforms:



6. Phasor Diagram



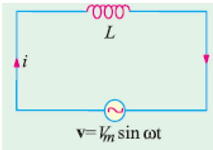
7. Opposition to current

V= I.R
I= V/R

Opposition is resistance of the resistor(R)

8. Power:

AC applied to Inductance only



$v = V_m \sin(\omega t)$

v= Ldi/dt
i = 1/L $\int v \cdot dt$

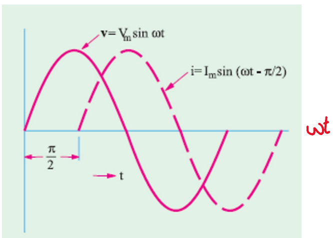
$i = \frac{V_m}{\omega L} \cdot (-\cos \omega t)$

$= I_m \sin(\omega t - \pi/2)$

Phasor Relation between V and I:

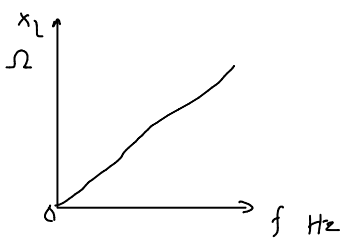
It is seen from equations that voltage and current are out-of-phase and phase difference is phi = Pi/2

Current is lagging the voltage by 90 degrees or pi/2 rad.

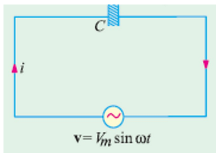


V = I. wL
XL =wL
V =I. XL   I= V/XL
Where XL is the opposition to current by inductance.
XL is called as Inductive Reactance
Its unit is Ohm if w is in rad/s and L is in Henry

XL = w.L = 2.π.f.L Ω
XL ∝ f



AC applied to Capacitance only



$v = V_m \sin(\omega t)$

i= Cdv/dt

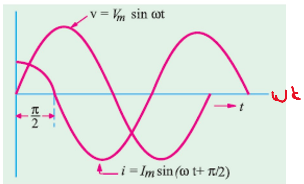
$i = \frac{V_m}{\frac{1}{\omega C}} \cdot \sin(\omega t + \pi/2)$

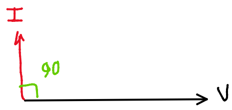
$= I_m \sin(\omega t + \pi/2)$

Phasor Relation between V and I:

It is seen from equations that voltage and current are out-of-phase and phase difference is phi = Pi/2

Current is leading the voltage by 90 degrees or pi/2 rad.

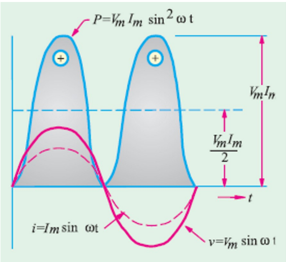
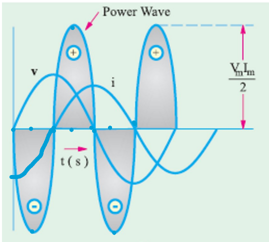
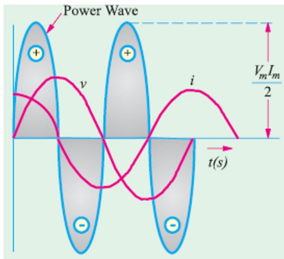




V = I. 1/wC
Xc =1/wC
V =I. Xc   I= V/Xc
Where Xc is the opposition to current by capacitance.
Xc is called as Capacitive Reactance
Its unit is Ohm if w is in rad/s and C is in Farad

Xc = 1/ w.C = 1/2.π.f.C

Xc ∝ 1/f

p= v.i = Vmsin(w.t). Im sin(w.t) = Vm Im $\sin^2$(w.t) = Vm. Im /2 - Vm Im cos(2w.t)/2 Average power over complete cycle P = Vm .Im/2 P = V. I Watt 	P= 0 A pure inductance doesn't consume any power. OR Average Power consumed by a pure inductance is zero 	P= 0 A pure capacitance doesn't consume any power. Average Power consumed by a pure inductance is zero 

Examples

1. A 60-Hz voltage of 115 V (r.m.s.) is impressed on a 100 ohm resistance :

(i) Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at t = 0 (ii) Show the voltage and current on a time diagram. (iii) Show the voltage and current on a phasor diagram Find the RMS value of current and power consumed