

D.C. Circuit

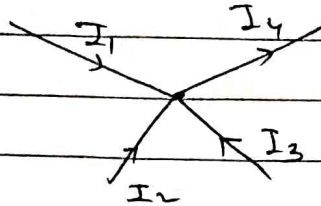
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* Kirchhoff's Current law

- 1) The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.
- 2) Total incoming current is equal to total outgoing current.

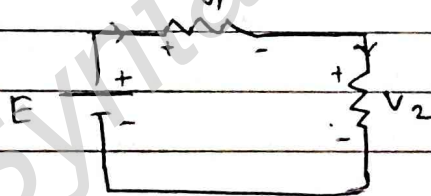


$\therefore$ KCL eqn is

$$I_1 + I_2 + I_3 - I_4 = 0$$

* Kirchhoff's voltage law

In any closed loop of electric circuit, the algebraic sum of the electromotive forces and voltage drops is equal to zero.



KVL eqn is

$$E - V_1 - V_2 = 0$$

* Superposition Theorem

In any linear bilateral active network containing more than one source, the current in any branch is the sum of current in that branch when each source is acting separately while all other sources are replaced by their internal resistance.

* Thevenin's theorem

In any linear bilateral Active network between given two point can be replaced by its equivalent consisting of single voltage source and series resistance.

The voltage source is equal to open circuit voltage between given two terminals and the series resistance is the equivalent resistance between two given terminals.

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

* Norton's theorem

Any two terminal network can be replaced by a single current source of magnitude I_N in parallel with a single resistance R_N .

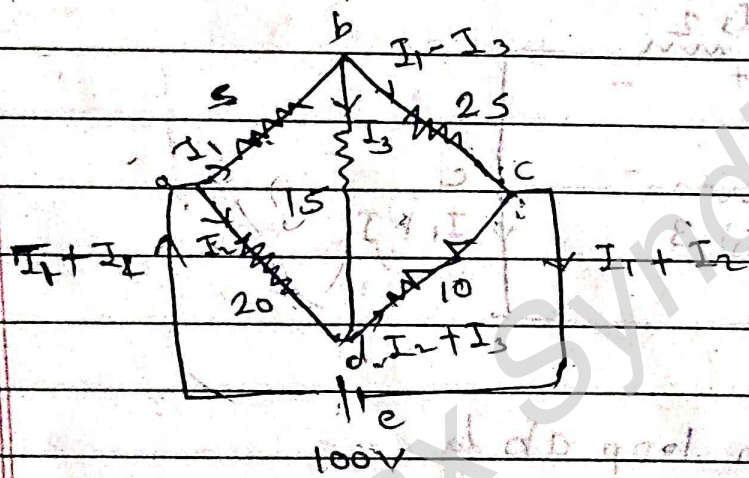
$$I_L = I_N \frac{R_N}{R_L + R_N}$$

* Maximum power transfer theorem.

Maximum power is transfer when load resistance R_L is equal to internal resistance of the source R_i i.e. the condition for maximum power transfer is $R_L = R_i$

$$P_{max} = \frac{V_s^2}{4R_i}$$

Find the current through each branch using KCL / KVL each.



For loop abda, $I_1 + I_2 - I_3 -$
 $- I_1 5 - 15 I_3 + 20 I_2 = 0 \quad \text{--- (1)}$

For loop bcdb, $I_1 - I_3 + I_2 -$
 $- 25(I_1 - I_3) + 10(I_2 + I_3) + 15 I_3 = 0$

$$-25I_1 + 25I_3 + 10I_2 + 10I_3 + 15I_3 = 0$$

$$-25I_1 + 10I_2 + 50I_3 = 0 \quad \text{--- (1)}$$

adc. ~~da~~

$$-I_1 + I_2 - 20I_2 - 10I_2 + 10I_3 - I_1 + I_2 + 100 = 0$$

$$-2I_1 - 32I_2 - 10I_3 + 100 = 0 \quad \text{--- (2)}$$

$$-20I_2 - 10I_2 - 10I_3 + 100 = 0$$

$$-30I_2 - 10I_3 + 100 = 0 \quad \text{--- (3)}$$

$$I_1 = 4.946A \quad I_2 = 2.688A \quad I_3 = 1.935A$$

$I_1 = 4.94A \rightarrow$ current through 5Ω

$I_2 = 2.688A \rightarrow$ current through 20Ω

$I_3 = 1.935A \rightarrow$ current through 15Ω

Current through $10\Omega = I_2 + I_3 = 2.68 + 1.93$

$$= 4.61A$$

Current through $25\Omega = I_1 - I_3 = 4.94 - 1.93A$

$$= 3.01A$$

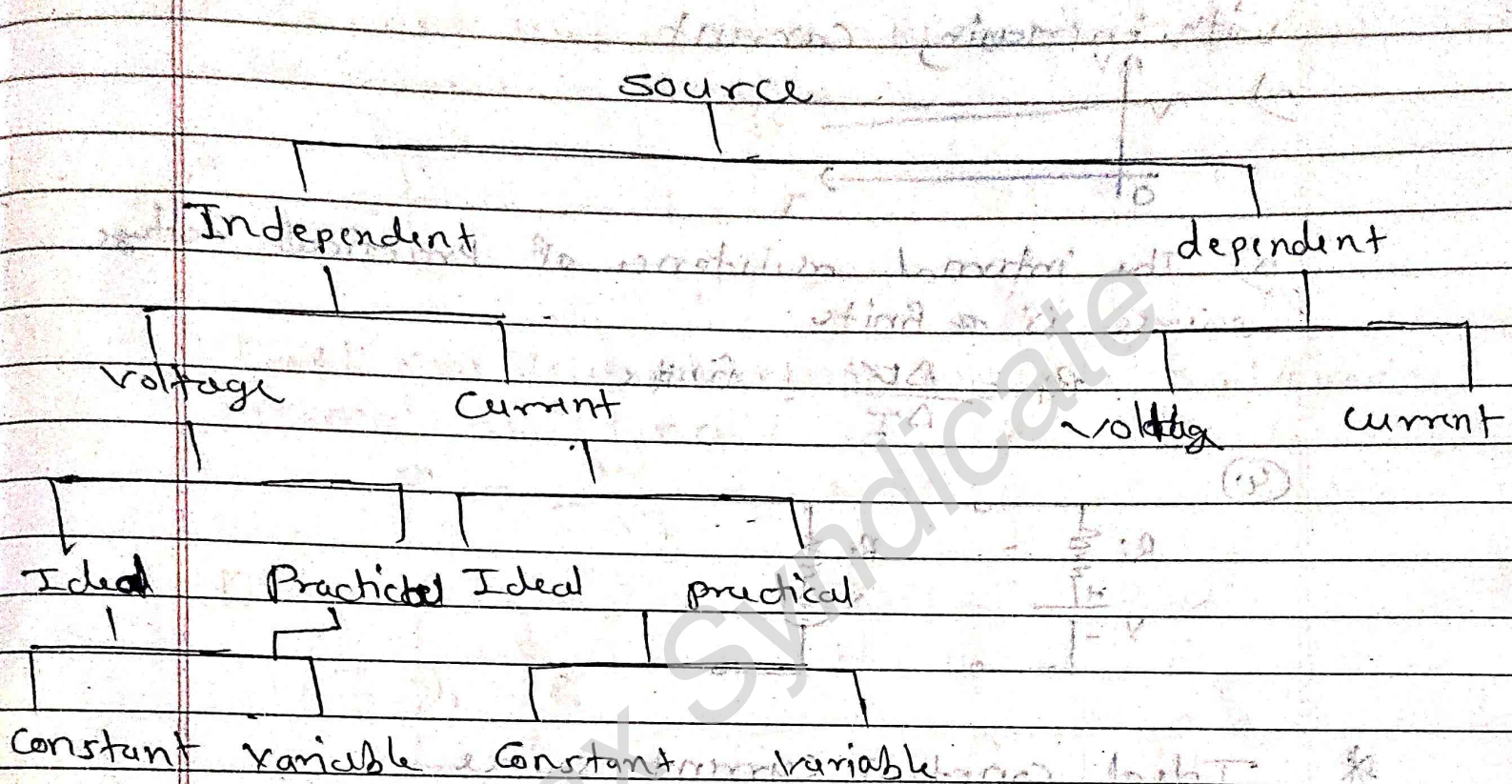
Current through battery $= I_1 + I_2 = 4.94 + 2.688$

$$= 7.62A$$

$$0 = 10V + 2.0 \times 0.0 - 0 \times 0.01$$

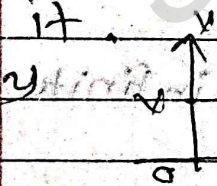
$$[V_{00} = 10V]$$

* Types of Source and source transformation



* Ideal constant voltage source

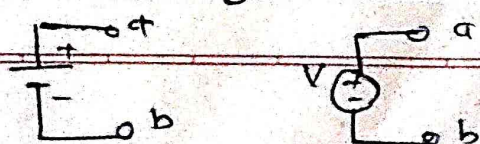
1) The source for which voltage remain constant for any value of current drawn from it.



2) The internal resistance of Ideal voltage source is zero

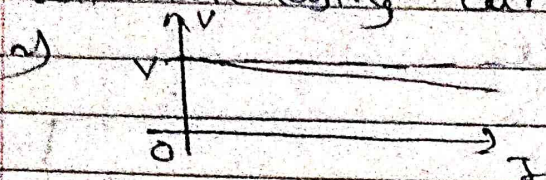
$$R_i = \frac{\Delta V}{\Delta I} = 0$$

3) an Ideal voltage source is represented by



* Practical

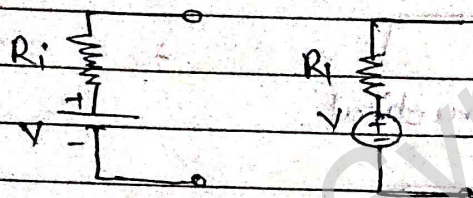
1) A source for which the voltage decreases with increasing current.



3) The internal resistance of Practical voltage source is finite

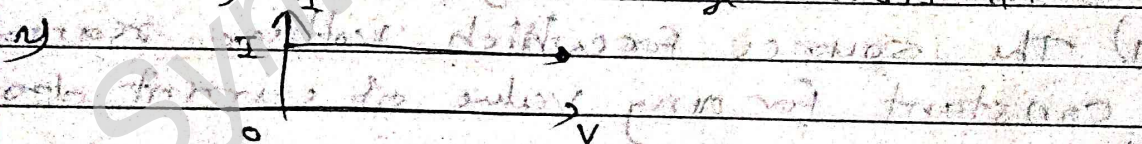
$$R_i = \frac{\Delta V}{\Delta I} = \text{finite}$$

4)



* Ideal Constant Current source

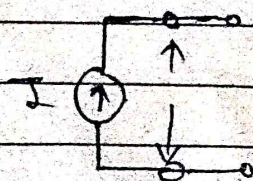
1) A source for which current remains constant for any value of voltage across it.



3) Ideal current source has infinite internal resistance.

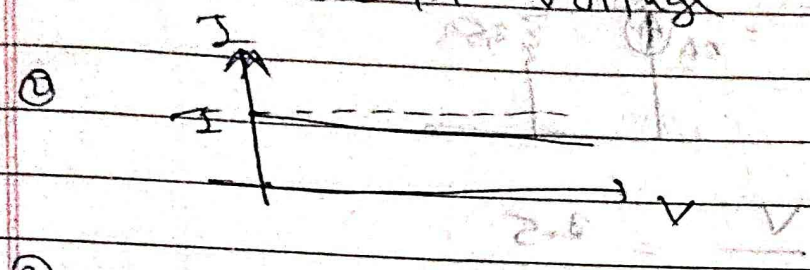
$$R_i = \frac{\Delta V}{\Delta I} = \infty$$

4)



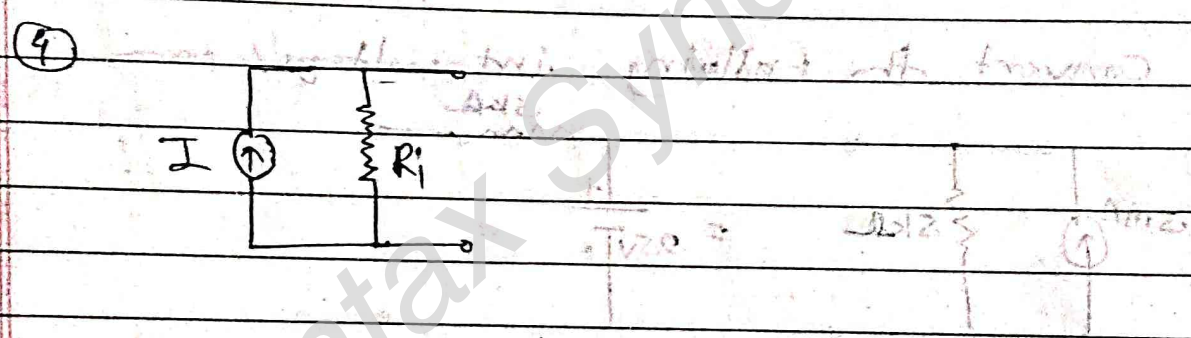
Practical

① A current source for which current decreases with increase in voltage



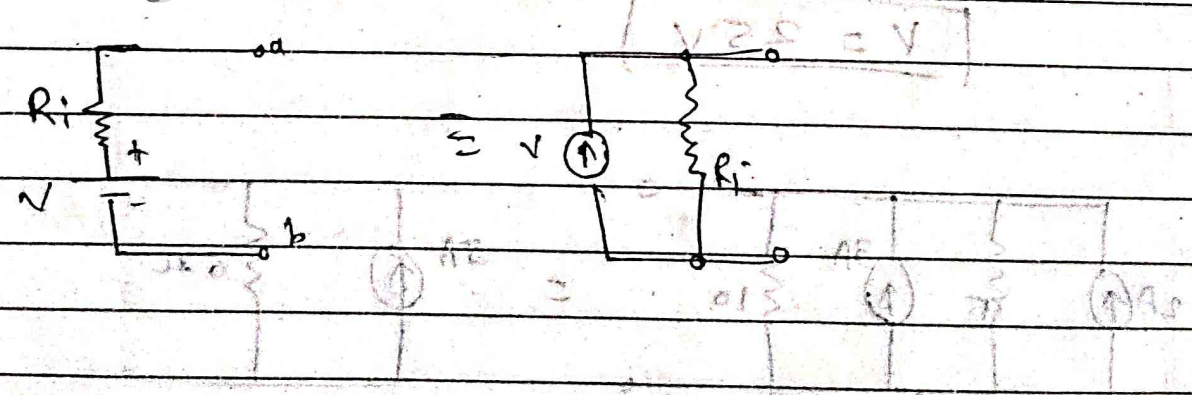
③ A practical current source has finite internal resistance

$$R_i = \frac{\Delta V}{\Delta I} = \text{finite}$$

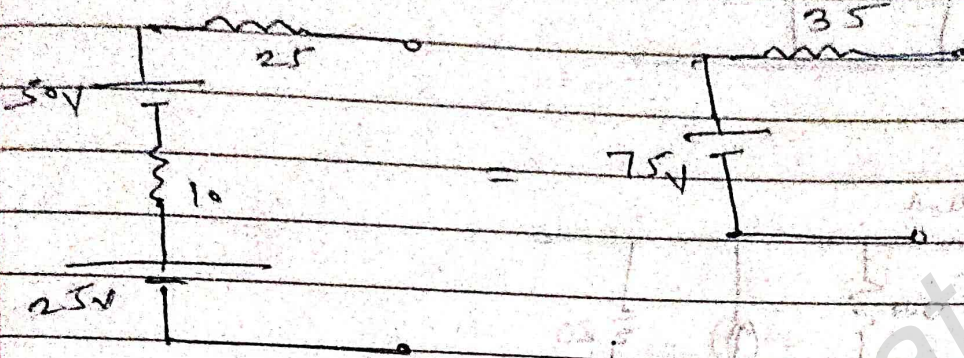


* Source Transformation

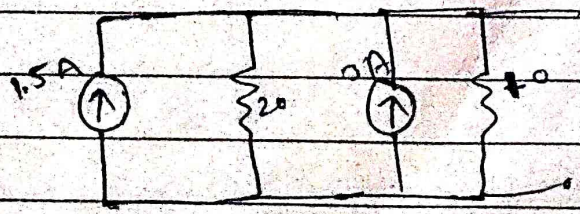
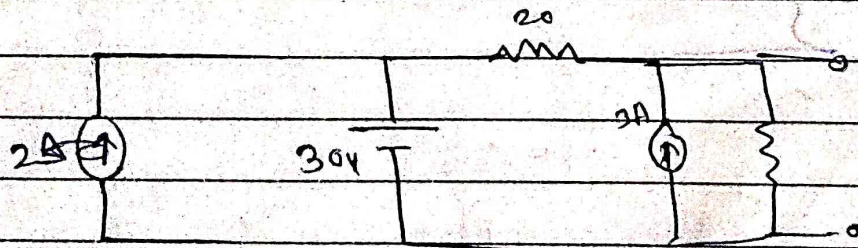
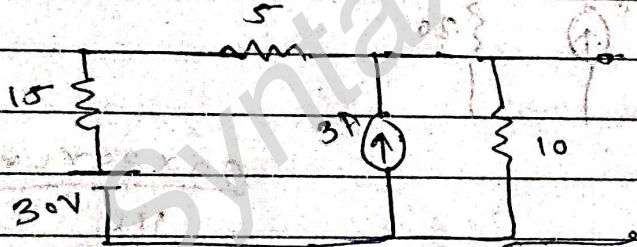
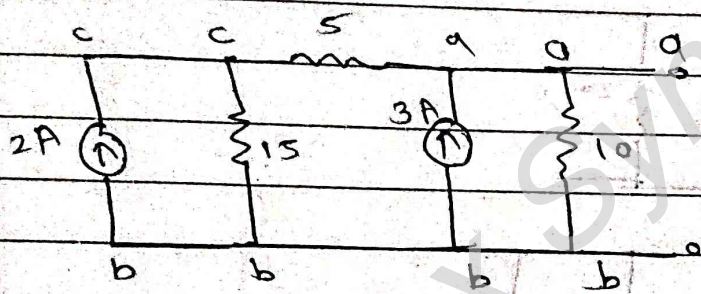
voltage source $\rightarrow$ current source



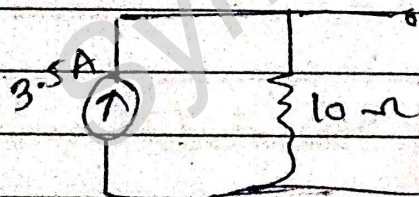
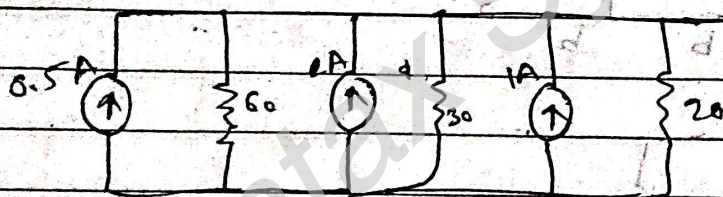
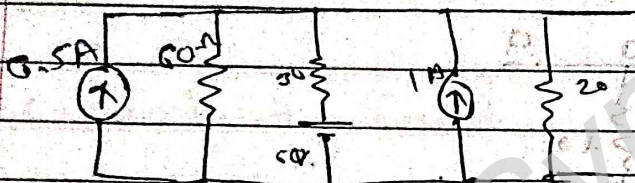
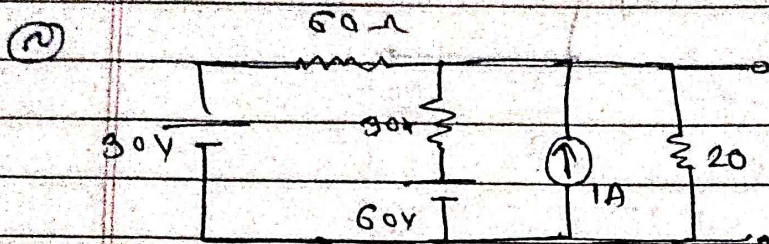
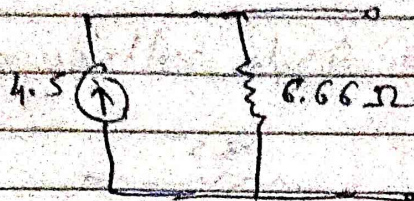
Ques Convert the following into single voltage source and series resistance



Q1



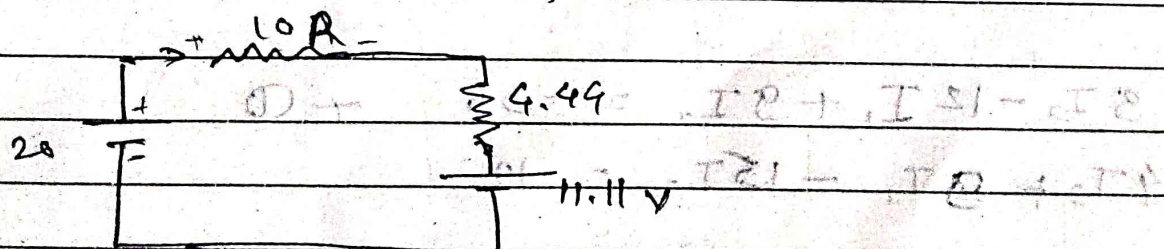
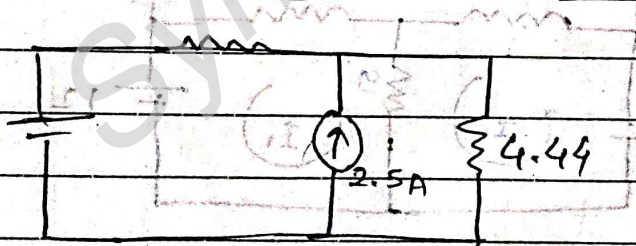
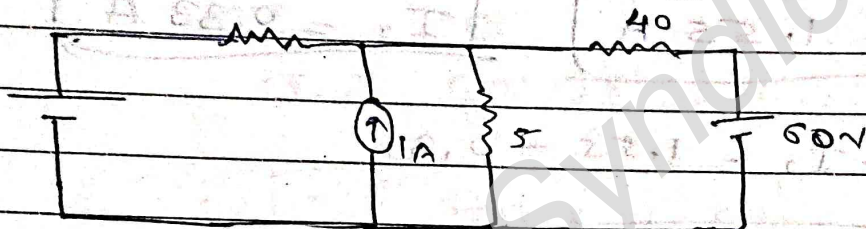
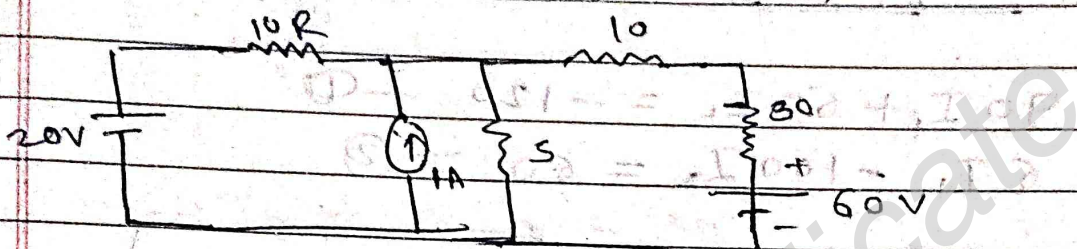
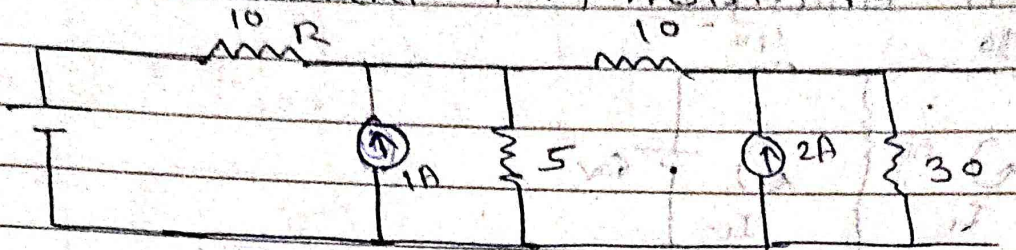
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



$$\frac{60 \times 30 \times 20}{118} = 118 \text{ V}$$

Note - do not convert the resistor in which current is to be found out in voltage or current source.

Find current from resistor R



$$20 - 10 \times I - 4.4 I - 11 = 0$$

$$9 - 14.4 I = 0$$

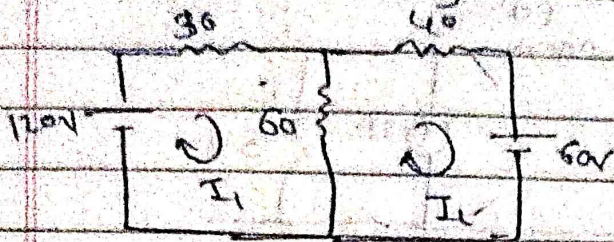
$$-14.4 I = -9$$

$$I = 0.625 A$$

$$I_{EX} = 1.55$$

$$I_{EX} = 0.333$$

* Mesh Analysis



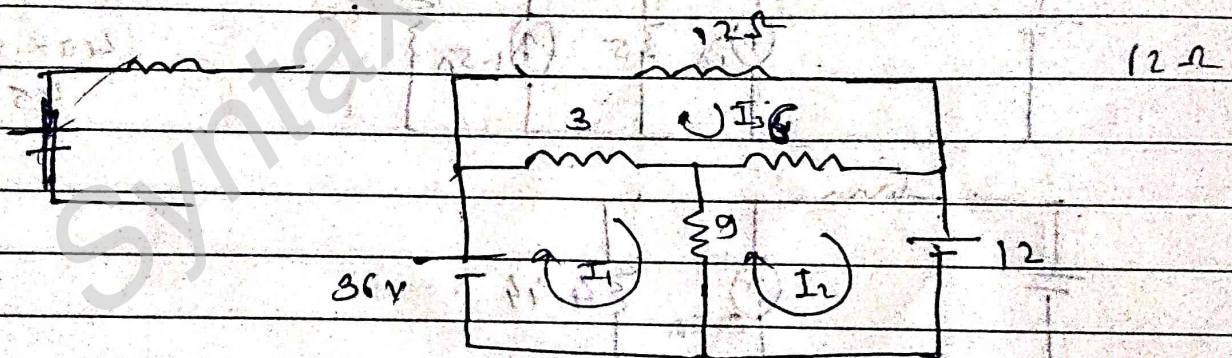
$$-90I_1 + 60I_2 = -120 \quad \text{--- (1)}$$

$$6I_1 - 100I_2 = 60 \quad \text{--- (2)}$$

$$I_1 = 1.55 \text{ A} \quad I_2 = 0.33 \text{ A}$$

$$I_{60} = I_1 - I_2 = 1.55 - 0.33$$

$$I_{60} = I_1 - I_2 = 1.22 \text{ A}$$

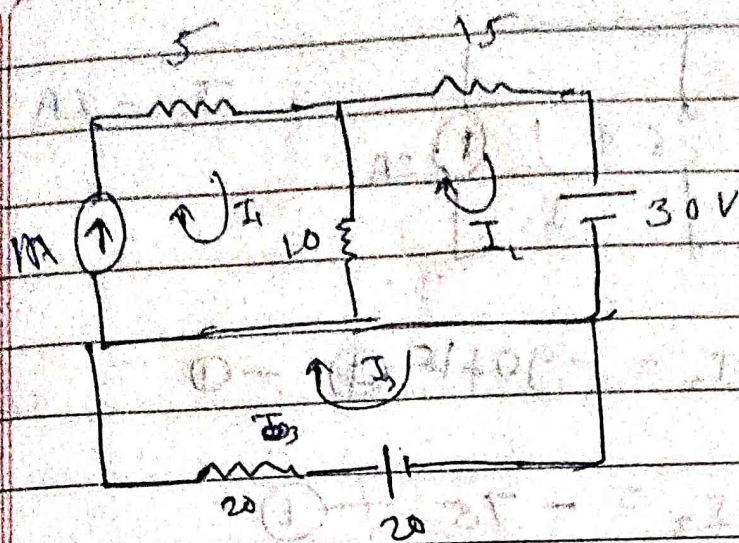


$$3I_1 - 12I_2 + 9I_3 = -36 \quad \text{--- (1)}$$

$$4I_1 + 3I_2 - 15I_3 = 12$$

$$-21I_1 + 3I_2 + 6I_3 = 0$$

$$I_3 = 2 \text{ A}, \quad I_1 = 6 + 36, \quad I_2 = 3.81 \text{ A}$$



$$-25I_2 + 10I_1 = 30$$

$$-25I_2 = 30$$

$$I_2 = \frac{30}{-25} = -1.2$$

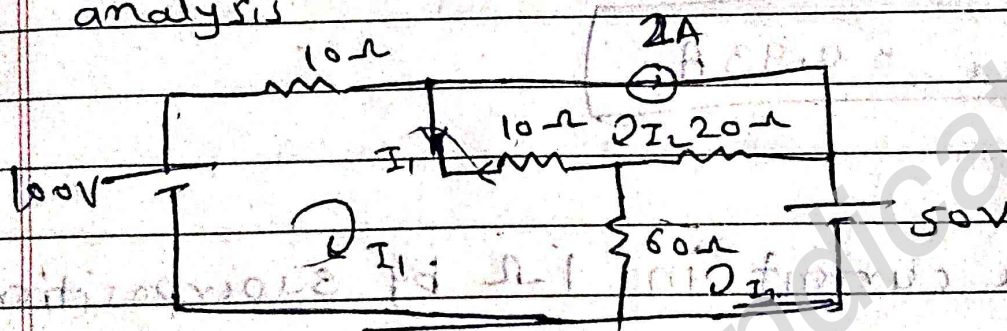
$$\boxed{I_2 = -1.2 \text{ A}}$$

$$-20I_3 = -20$$

$$\boxed{I_3 = 1 \text{ A}}$$

7M

Q. Find the current through 60Ω by mesh analysis



$$-10I_1 - 10(I_1 - I_2) - 60(I_1 - I_3) + 100 = 0$$

$$-10I_1 - 10I_1 + 10I_2 - 60I_1 + 60I_3 = -100$$

$$-80I_1 + 10I_2 + 60I_3 = -100 \quad \text{--- (1)}$$

$$I_2 = 2A$$

$$-80I_1 + 20 + 60I_3 = -100$$

$$-80I_1 + 60I_3 = -120 \quad \text{--- (2)}$$

$$-20(I_3 - I_2) - 50 + 60(I_3 - I_1) = 0$$

$$-20I_3 + 20I_2 - 50 - 60I_3 + 60I_1 = 50$$

$$60I_1 - 80I_3 + 40 = 50$$

$$60I_1 - 80I_3 = 10 \quad \text{--- (2)}$$

from eqn (1) and (2) $V_{II} = 4V$

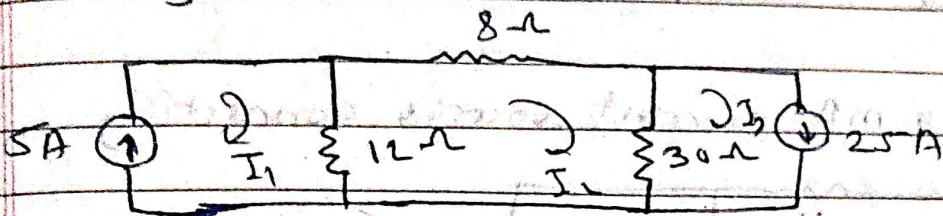
$$\boxed{I_1 = 3.21 A} \quad V_{II} = 4V$$

$$\boxed{I_3 = 2.28 A} \quad V_{II} = 4V$$

$$I_{\text{scd}} = I_1 - I_3 = 3.21 - 2.28 = 0.93 A \downarrow$$

$$\boxed{I_{\text{scd}} = 0.93 A}$$

5) Find the current through 8Ω by mesh analysis



$$I_1 = 5A \quad \text{---} \textcircled{1}$$

$$I_3 = 25A \quad \text{---} \textcircled{2}$$

$$-8I_2 - 12(I_2 - I_1) - 30(I_2 - I_3) = 0$$

$$-8I_2 - 12I_2 + 12I_1 - 30I_2 + 30I_3 = 0$$

$$50I_2 = 12I_1 + 30I_3$$

$$50I_2 = 12 \times 5 + 30 \times 25$$

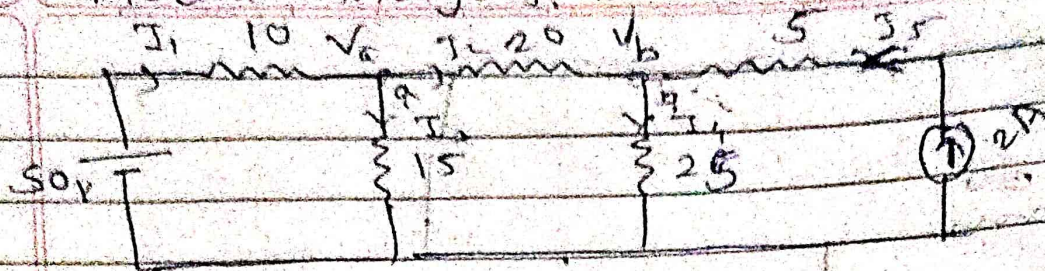
$$I_2 = \frac{810}{50}$$

$$I_2 = 16.2A$$

$$I_{8\Omega} = I_2$$

$$I_{8\Omega} = 16.2A$$

Nodal Analysis



Find current through 20Ω

apply kcl at node a

$$I_1 - I_2 - I_3 = 0 \quad \text{--- (1)}$$

at node b

$$I_2 - I_4 + I_5 = 0 \quad \text{--- (2)}$$

$$I_1 = \frac{50 - V_a}{10}$$

$$I_2 = \frac{V_a - V_b}{20}$$

$$I_3 = \frac{V_a - 0}{15}$$

$$I_4 = \frac{V_b - 0}{25}$$

$$I_5 = 2A$$

$$\frac{50 - V_a}{10} = \frac{V_a - V_b}{20} + \frac{V_a}{15} = 0$$

$$6(50 - V_a) - 3(V_a - V_b) - 4V_a = 0$$

$$300 - 6V_a - 3V_a + 3V_b - 4V_a = 0$$

$$+ \bullet 13V_a \neq 3V_b = 300 \quad \text{--- (3)}$$

$$\frac{(V_a - V_b)}{20} - \frac{V_b}{25} + 2A = 0$$

$$\frac{5(V_a - V_b) - 4V_b + 200}{100} = 0$$

$$5V_a - 5V_b - 4V_b + 200 = 0$$

$$5V_a - 9V_b = -200 \quad \text{--- (2)}$$

$$V_a = 19.32$$

$$V_b = 16.28$$

$$V_a = 27.94$$

$$V_b = 21.07$$

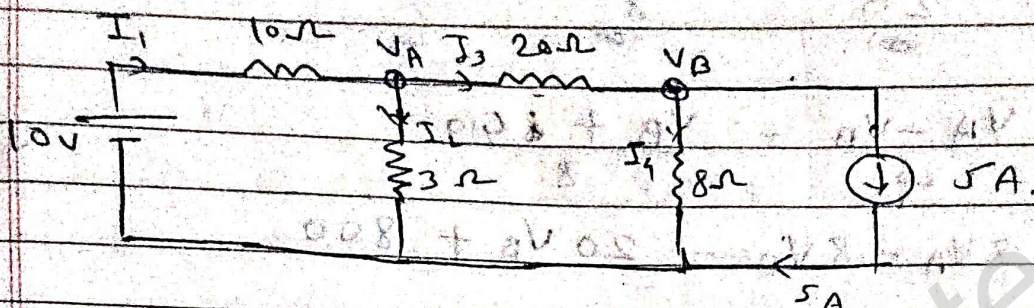
I_{20}

$$\boxed{V_a = 32.35}$$

$$\boxed{V_b = 40.19}$$

4M

① Find Nodal voltage V_A and V_B



At node A

$$I_1 = I_2 + I_3 \quad \text{--- (1)}$$

at node B

$$I_3 = I_4 + I_5$$

$$I_1 = \frac{10 - V_A}{10} \quad I_2 = \frac{V_A}{3} \quad I_3 = \frac{V_A - V_B}{20}$$

$$I_4 = \frac{V_B}{8} \quad I_5 = 5$$

$$\frac{10 - V_A}{10} = \frac{V_A}{3} + \frac{V_A - V_B}{20}$$

$$\frac{10 - V_A}{10} = \frac{20V_A + 3V_A - 3V_B}{60}$$

$$60 - 6V_A = 23V_A - 3V_B$$

$$29V_A - 3V_B = 60 \quad \text{--- (2)}$$

$$\frac{V_A - V_D}{20} = \frac{V_B}{8} + 1.5 \text{ A}$$

$$V_A - V_D = \frac{V_B}{8} + 12$$

$$8V_A - 8V_D = V_B + 96$$

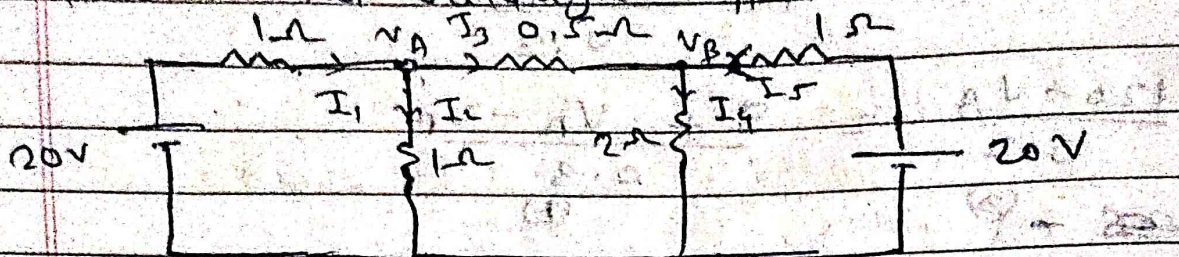
$$8V_A - 28V_B = 96 \quad \text{--- (4)}$$

$$V_A = -0.91 \text{ V}$$

$$V_B = -28.83 \text{ V}$$

⑤

Find Nodal voltage V_A and V_B



At node V_A $AVS + 81 = AVS - 0.088$

$$I_1 = I_2 + I_3 \quad \text{--- (1)}$$

At node V_B $AVS + 81 = AVS - 0.088$

$$I_4 = I_3 + I_5 \quad \text{--- (2)}$$

$$81 - 0.088 = AVS - AV2$$

$$I_1 = \frac{20 - V_A}{1} \quad \text{and} \quad I_2 = \frac{V_A - V_B}{0.5}$$

$$I_3 = \frac{V_A - V_B}{0.5} \quad \text{and} \quad I_4 = \frac{V_B}{2} = 4V$$

$$I_5 = \frac{20 - V_B}{1} = 20 - V_B$$

eqn (1) becomes $AVS - 0.088 + AV - 4V$

$$20 - V_A = \frac{V_A - V_B}{0.5} + 20 - V_B \quad \text{--- (3)}$$

$$20 - V_A = V_A + 2V_B - 2V_B \quad \text{--- (3)}$$

$$4V_A - 2V_B = 20 \quad \text{--- (3)}$$

eqn (2) becomes

$$\frac{V_B}{2} = \frac{V_A - V_B}{0.5} + 20 - V_B$$

$$\frac{V_B}{2} = 2V_A - 2V_B + 20 - V_B$$

$$\frac{V_B}{2} = 2V_A - 3V_B + 20$$

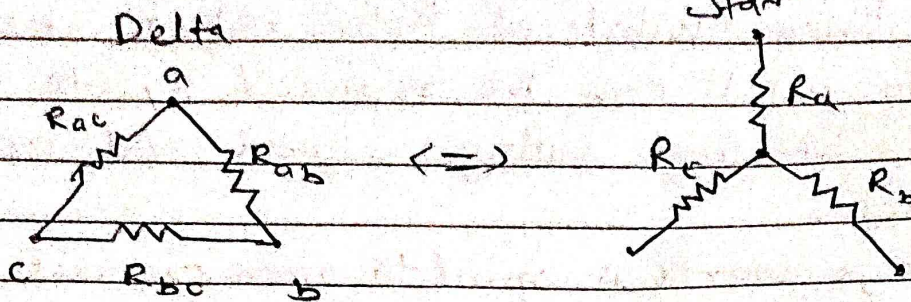
$$V_B = 4V_A - 6V_B + 40$$

$$4V_A - 7V_B = -40 \quad \text{--- (4)}$$

$$V_A = 11 \text{ V}$$

$$V_B = 12 \text{ V}$$

* Star / Delta transformer



$$R_a = \frac{R_{ca} R_{ab}}{R_{ac} + R_{ab} + R_{bc}}$$

$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c}$$

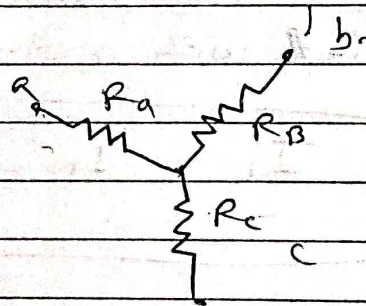
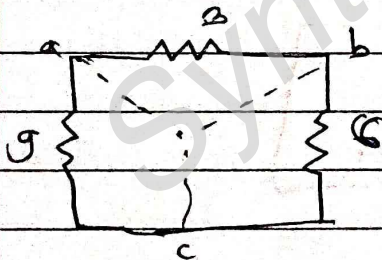
$$R_b = \frac{R_{bc} R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a}$$

$$R_c = \frac{R_{ac} \cdot R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_{ac} = R_a + R_c + \frac{R_a R_c}{R_b}$$

Convert into Star



$$R_a = \frac{3 \times 6}{3 + 6 + 9} = \frac{3 \times 6}{18} = \frac{27}{18}$$

$$R_a = 1.5 \Omega$$

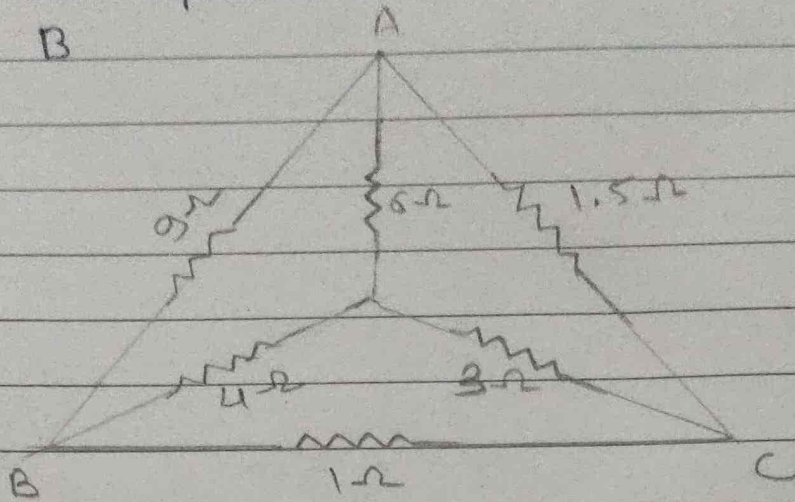
$$R_c = \frac{6 \times 9}{3 + 9 + 6} = \frac{54}{18}$$

$$R_c = 3$$

$$R_b = \frac{3 \times 9}{3 + 9 + 6}$$

$$R_b = 1$$

Find an equivalent resistance between terminals A and B



Convert star into delta.

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

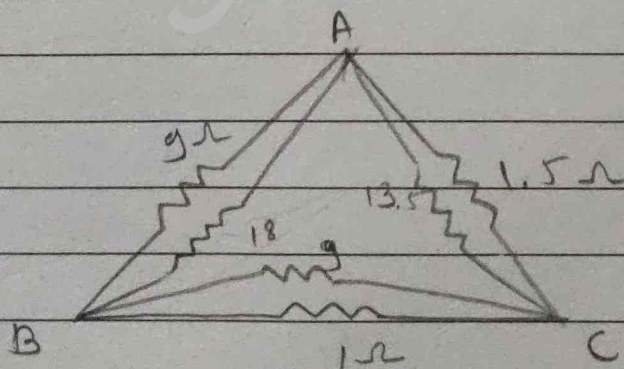
$$R_{AB} = 6 + 4 + \frac{6 \times 4}{3} = 18 \Omega$$

$$R_{BC} = 4 + 3 + \frac{4 \times 3}{6}$$

$$R_{BC} = 9 \Omega$$

$$R_{AC} = 6 + 3 + \frac{6 \times 3}{4}$$

$$R_{AC} = 13.5 \Omega$$

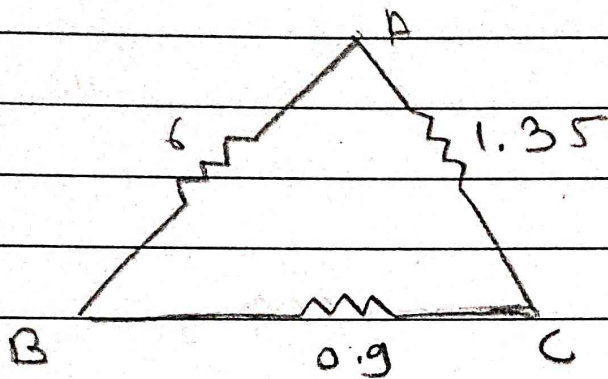


$$R_{AB} = \frac{9 \times 18}{27} = 6 \Omega$$

$$R_{BC} = \frac{9 \times 1}{10} = 0.9 \Omega$$

$$R_{AC} = 13.5 \Omega$$

$$R_{AC} = \frac{13.5 \times 1.5}{15} = 1.35$$



$$R_{AB \text{ eq}} = R_{AB} \parallel (R_{BC} + R_{AC})$$

$$= \frac{6 \times (0.9 + 1.35)}{6 + (0.9 + 1.35)}$$

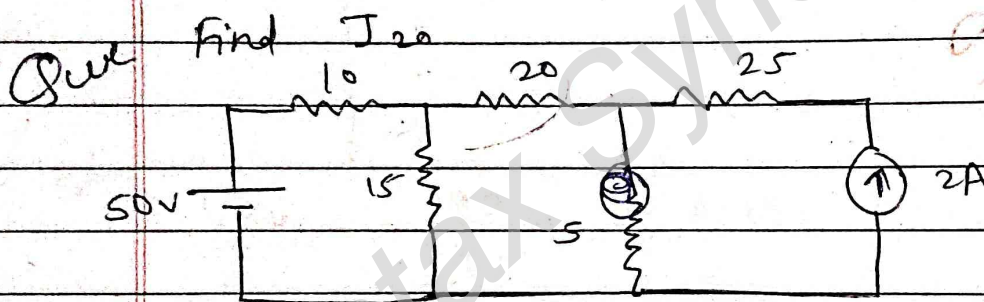
$$= \frac{13.5}{8.25}$$

$$R_{AB} = 1.636 \Omega$$

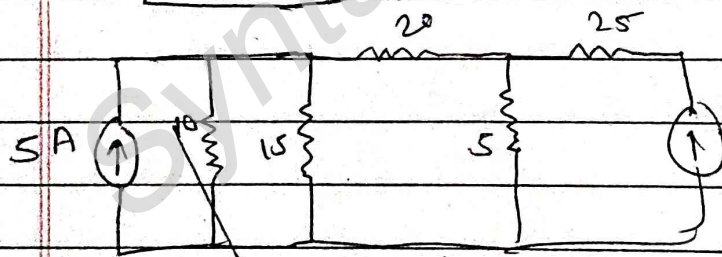
* Superposition Theorem

This theorem is applicable for linear networks only.

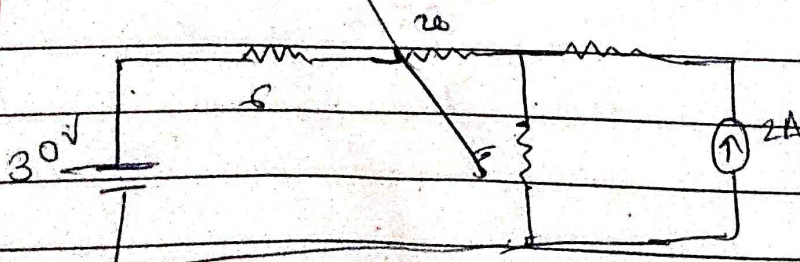
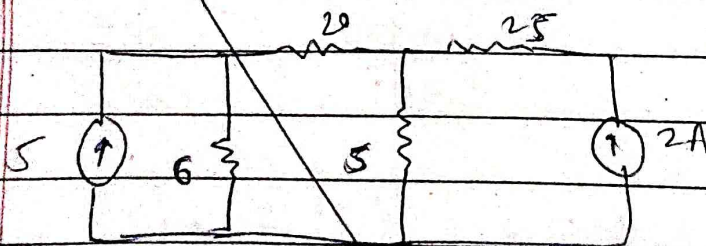
Statement - In any linear bilateral active network containing more than one source, the current in any branch is the sum of current in that branch when each source is acting separately, while all other sources are replaced by their internal resistance.



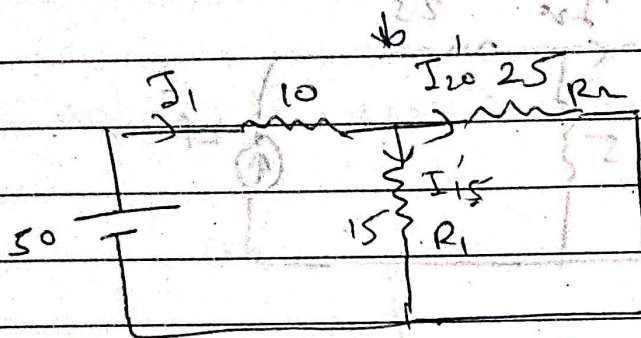
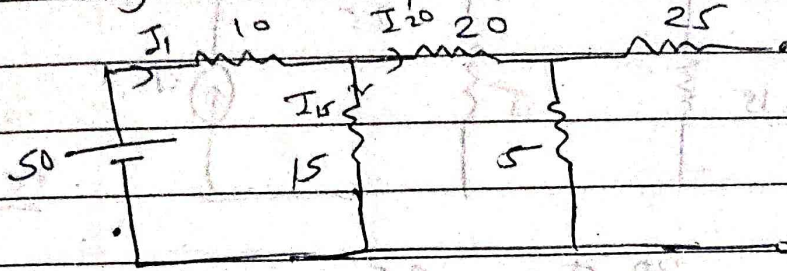
Find current through 20Ω



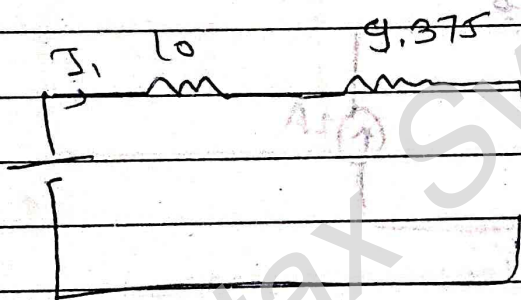
$$\frac{10 \times 15}{25} = 6A$$



① only 50V source connected

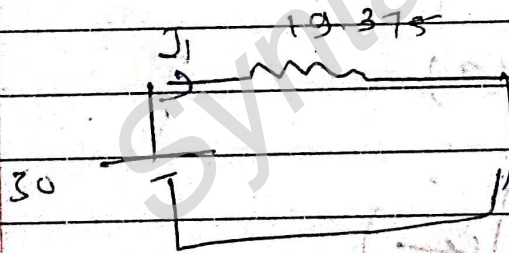


$$\frac{5 \times 5}{25 \times 15} = 9.375$$



$$I_{20} = \left(\frac{R_1}{R_1 + R_2} \right) I_1$$

$$I_{20} = \left(\frac{15}{40} \right) 2.58$$



$$I_{20} = 0.967 A$$

$$I_1 = \frac{50}{19.37} = 2.58 A$$

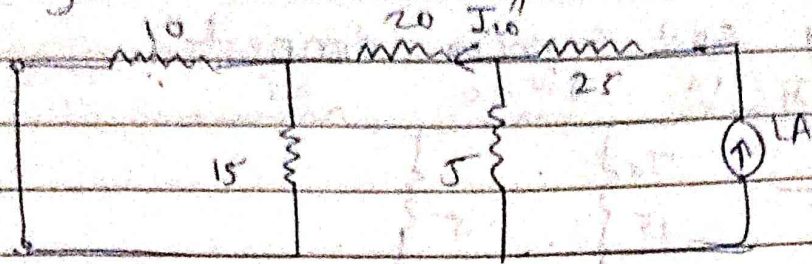
$$I_{20}' = \frac{15}{40} \times 2.58$$

$$\Rightarrow 0.967 A$$

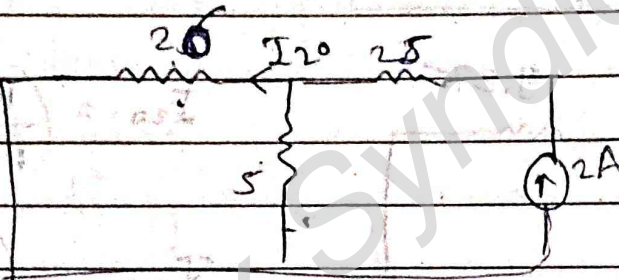
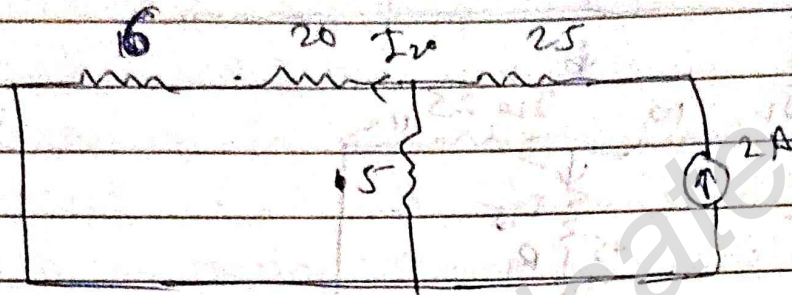
$$I_{20}' = 0.967 A$$

$$\left(\frac{P}{P_1 + P_2} \right)^2$$

only 2A source connected



$$10 \times 15$$



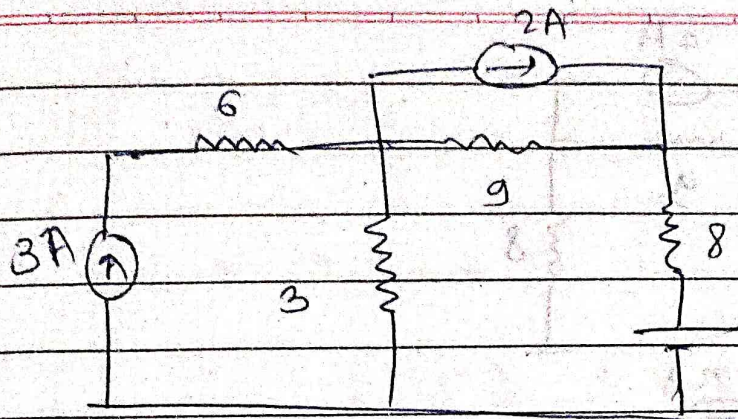
$$I''_{20} = \frac{5}{26 + 5} \quad (2)$$

$$I''_{20} = 0.32 \text{ A} \quad (\leftarrow)$$

$$I_{20} = I'_{20} + I''_{20}$$

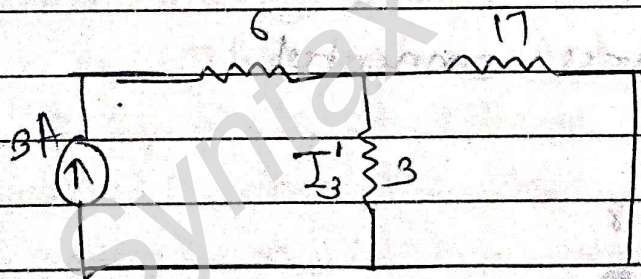
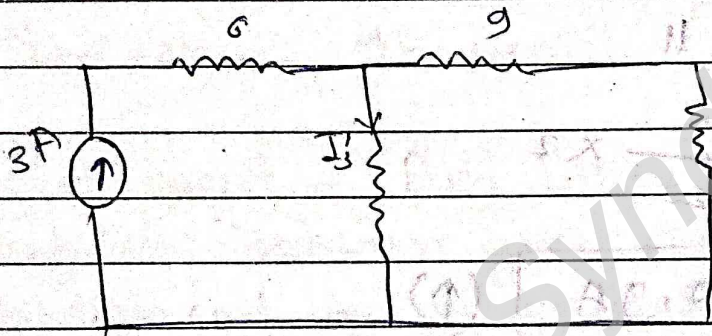
$$= 0.967 (\rightarrow) + 0.32 (\leftarrow)$$

$$I_{20} = 0.656 (\rightarrow) \text{ A}$$



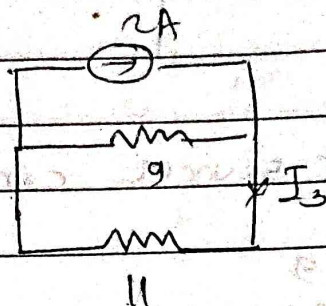
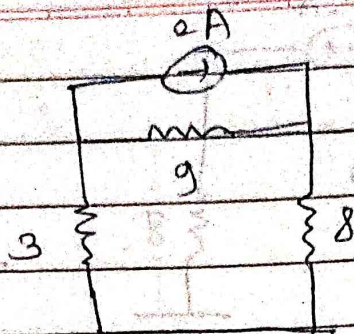
find current through 3 Ω

only 3A current source connected



$$I'_3 = \frac{17}{20} \times 3$$

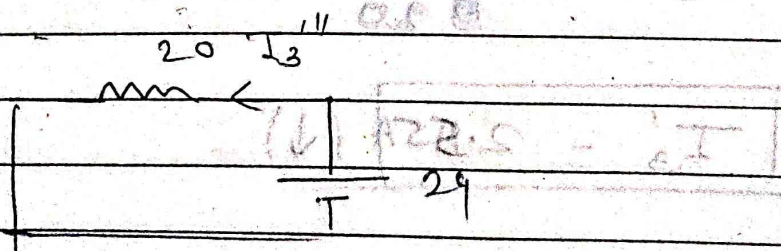
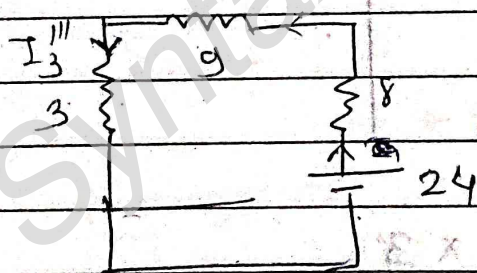
$$\boxed{I'_3 = 2.55A (\downarrow)}$$



$$I_3' = \frac{2}{20} \times 2$$

$$I_3' = 0.2 \text{ A} \quad (\uparrow)$$

only 24 V source connected



$$I_3 = \frac{24}{20 + 12}$$

$$I_3''' = 1.2 \text{ A} \quad (\downarrow)$$

$$I_3 = I_3'' + I_3'' + I_3'''$$

$$= 2.55 \downarrow + 0.9 \uparrow + 1.2 \downarrow$$

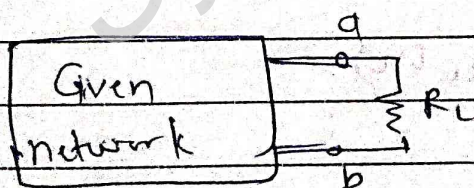
$$= 3.75 \downarrow + 0.9 \uparrow$$

$$\boxed{I_3 = 2.85 \downarrow A}$$

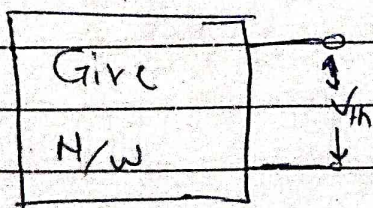
* Thevenin's Theorem

statement - Any linear Bilateral Active network between given two point can be replaced by its equivalent consisting of single voltage source and series resistance.

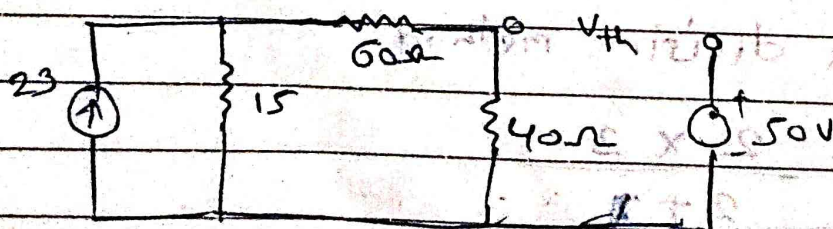
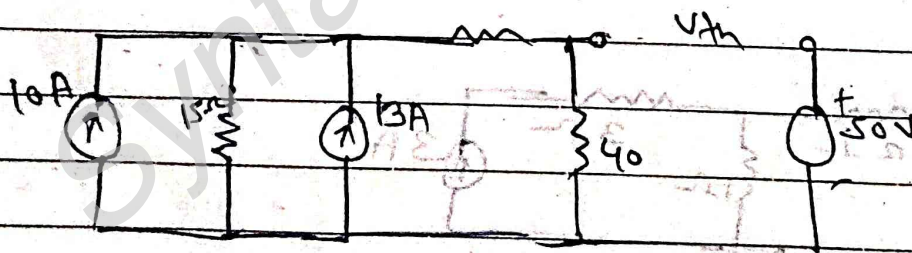
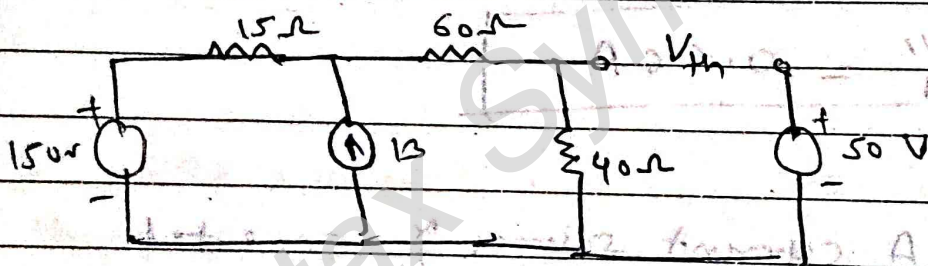
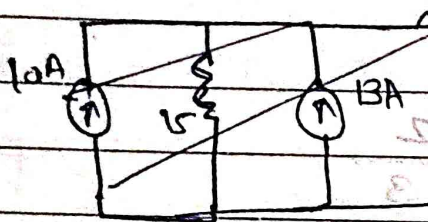
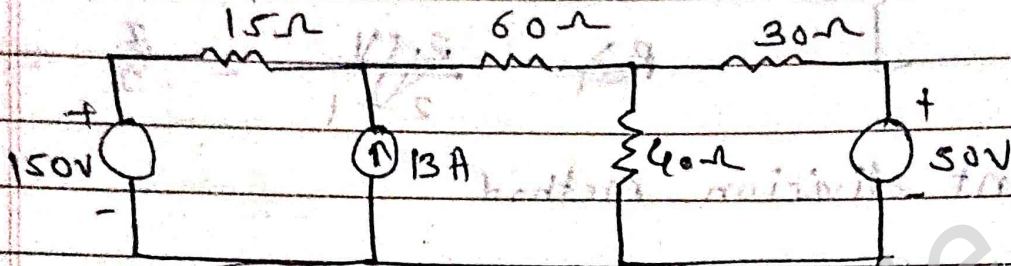
The value of voltage source is equal to open circuit voltage between given two terminals and the series resistance is the equivalent resistance between two given terminals.

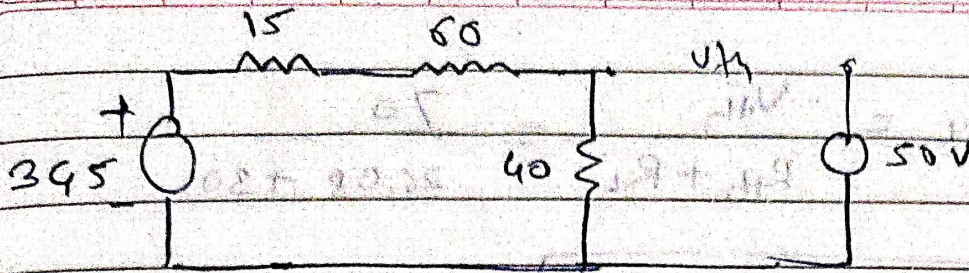


1) Finding V_{th}

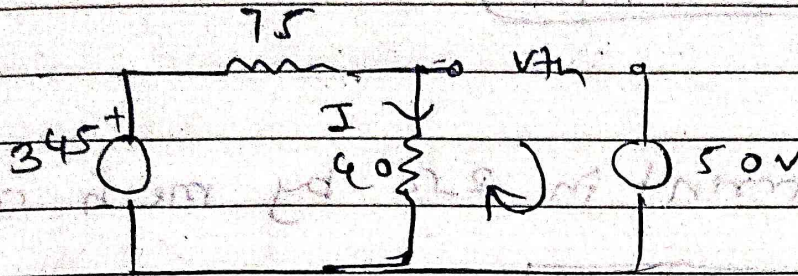


② Find the current through 30Ω by thevenin theorem





$A.P.S. I = I$



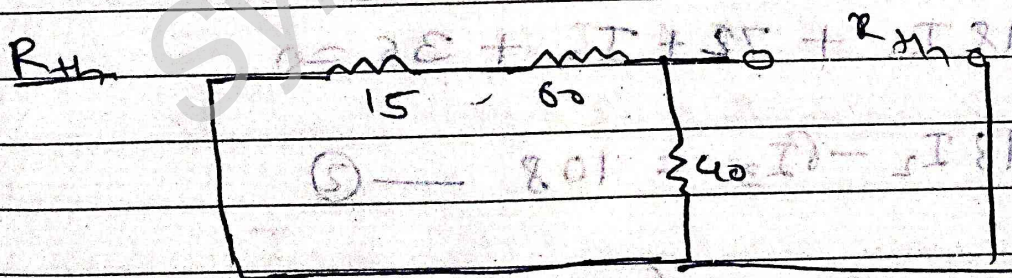
$$I = \frac{345}{115}$$

$$I = 3 \text{ A}$$

$$-V_{th} - 50 + 40I = 0$$

$$V_{th} = -50 + 40 \times 3$$

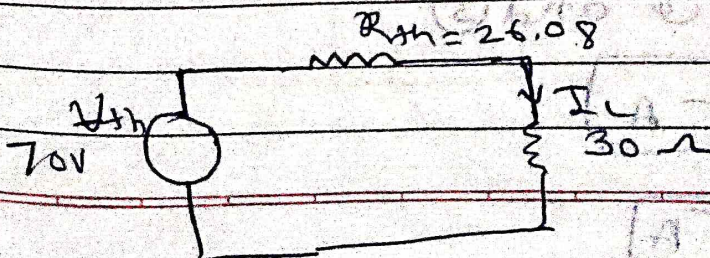
$$V_{th} = 70 \text{ V}$$



$$R_{th} = \frac{75 \times 40}{75 + 40}$$

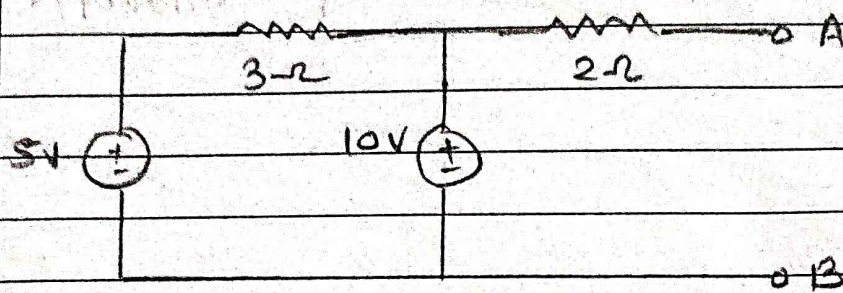
$$R_{th} = 26.08$$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{70}{26.08 + 30}$$

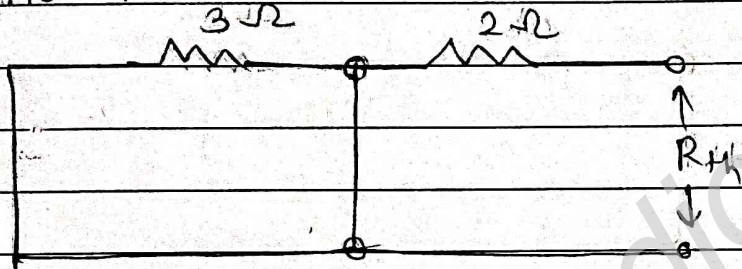


$$I_L = 1.24 \text{ A}$$

Q4) Find Thevenin's and Norton's equivalent circuit across A & B.

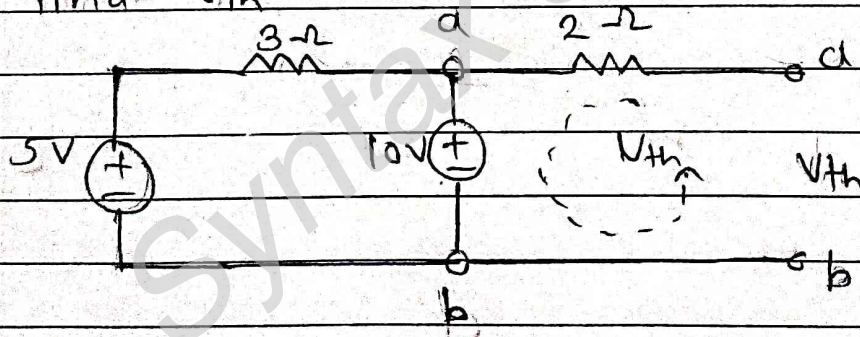


Find R_{th}



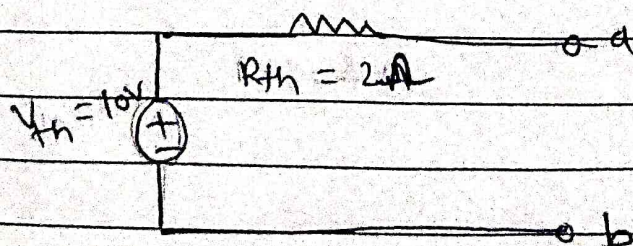
$$R_{th} = 2\Omega$$

Find V_{th}

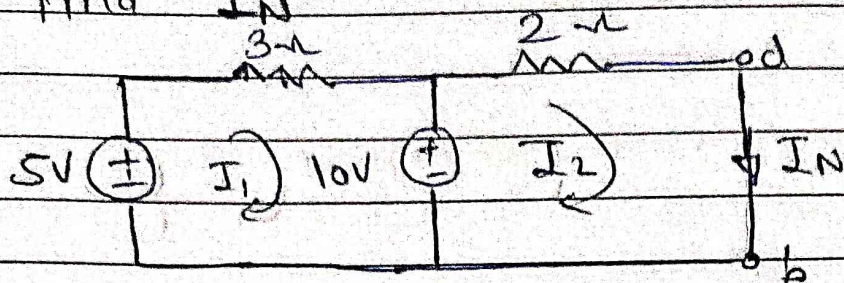


$$V_{th} = 10V$$

Thevenin's equivalent circuit is



Find I_N



by the mesh Analysis

$$-3I_1 - 10 + 5 = 0$$

$$I_1 = \frac{-5}{3}$$

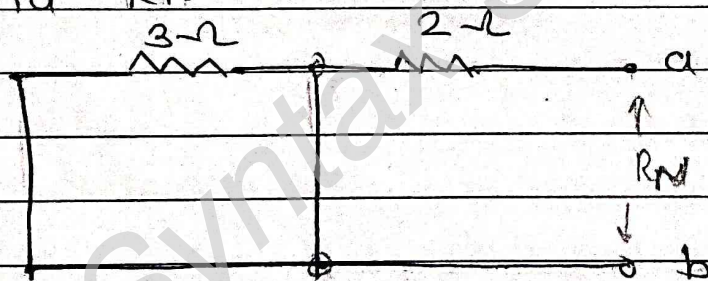
$$I_1 = -1.667 \text{ A}$$

$$-2I_2 + 10 = 0$$

$$I_2 = 5 \text{ A}$$

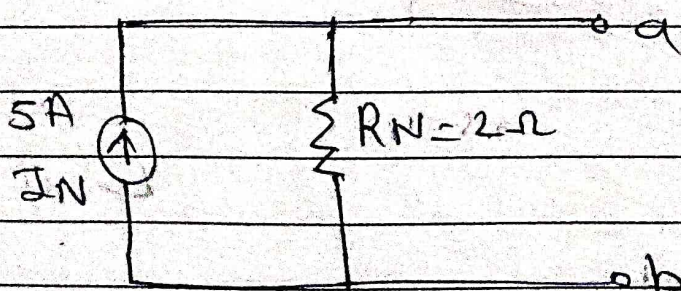
$$I_2 = I_N = 5 \text{ A}$$

Find R_N

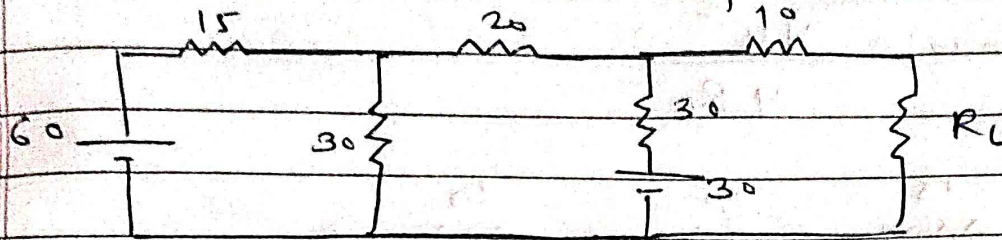


$$R_N = 2 \Omega$$

Norton equivalent circuit is

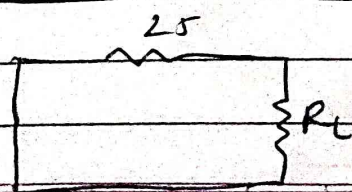
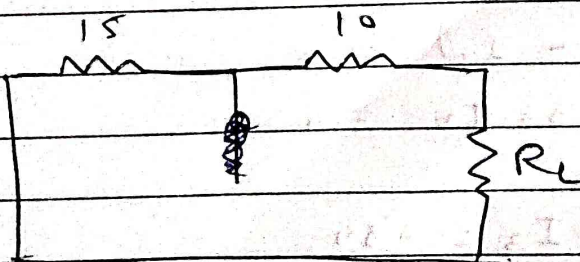
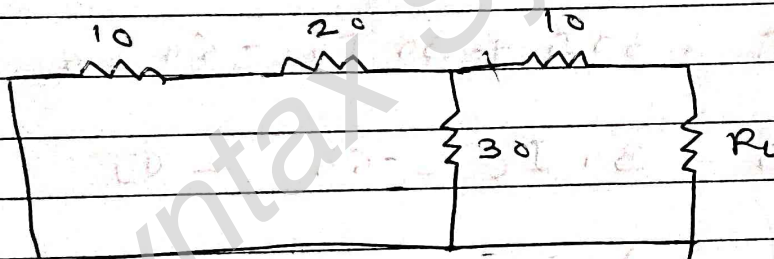


Ques Find the value of R_L for max power transfer
Find the value of max power



We know that for max power transfer

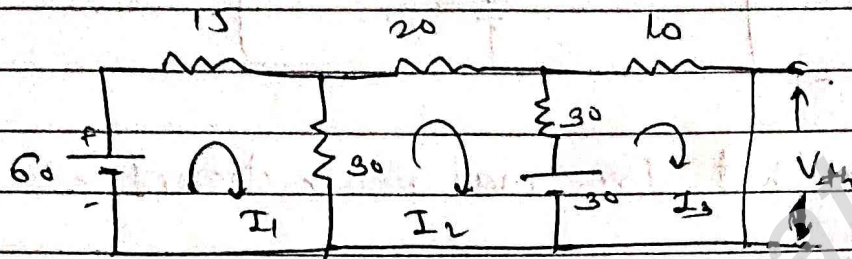
$$R_L = R_i$$



$$R_L = 25\Omega$$

Value of R_L for maximum power transfer is 25Ω

$$P_{\max} = \frac{V_{th}^2}{4R_L}$$



by mesh analysis

$$-60 - 15I_1 - 30(I_1 - I_2) = 0$$

$$-15I_1 - 30I_1 + 30I_2 - 60 = 0$$

$$45I_1 - 30I_2 = -60 \quad \text{--- (1)}$$

$$-30(I_2 - I_1) - 20I_2 - 30(I_2 - I_3) - 30 = 0$$

$$-30I_2 + 30I_1 - 20I_2 - 30I_2 + 30I_3 = 30$$

$$30I_1 - 80I_2 + 30I_3 = 30 \quad \text{--- (2)}$$

$$30 - 30(I_3 - I_2) = 0$$

$$30 - 30I_3 + 30I_2 = 0$$

$$30I_2 - 30I_3 = -30$$

$$I_1 = 1.44 \text{ A}$$

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$$I_3 = 0.1667 \text{ A}$$

$$I_2 = 1.1667 \text{ A}$$

$$-V_{th} + 30 + 0.1667 \times 10 = 0$$

$$V_{th} = 35 \text{ V}$$

$$P_{max} = \frac{V_{th}^2}{4 \times R_L}$$

$$= \frac{35^2}{4 \times 25}$$

$$P_{max} = 12.25 \text{ W}$$